Algebra 2/Pre-Calculus

Name__________________

More Exploration of Factoring and Division (Day 11, Polynomials)

In this problem set, we will investigate the connections between graphing, factoring, long division, and equation solving. A lot of these problems involve ideas from earlier in this unit, so don't hesitate to look back at your old handouts while you are working on this.

1. Consider the polynomial $x^3 - 6x^2 + 3x + 10$.

a. Does the polynomial $x^3 - 6x^2 + 3x + 10$ factor? Factor it, if you can.

b. None of the factoring methods that we have learned thus far are helpful in factoring $x^3 - 6x^2 + 3x + 10$. We need to use a different method. Graph $y_1 = x^3 - 6x^2 + 3x + 10$ on your calculator. Sketch the graph below.

c. Can we determine how $x^3 - 6x^2 + 3x + 10$ factors simply from its graph? ***Hint:*** Think about the x-intercepts.

Solution

The coordinates of the x-intercepts were $(-1,0)$, $(2,0)$, and $(5,0)$. Therefore, our polynomial has the following factors: $x+1$, $x-2$, and $x-5$. Hence, $x^3 - 6x^2 + 3x + 10 = (x+1)(x-2)(x-5)$.

2. Consider the polynomial $-2x^3 - 4x^2 + 38x + 40$.

a. Use your calculator to graph $y_1 = -2x^3 - 4x^2 + 38x + 40$. Then factor it.

b. Explain why $(x+5)(x+1)(x-4)$ is ***not*** the correct factoring. What ***is*** the correct factoring?

Answer: $-2(x+5)(x+1)(x-4)$

3. Consider the polynomial $x^3 - 5x^2 + 9x - 9$.

a. Can we use the method from the first two problems to factor $x^3 - 5x^2 + 9x - 9$? Explain. Factor it if you can.

b. If we graph $y_1 = x^3 - 5x^2 + 9x - 9$, we only get one x-intercept. Can you identify one of the factors of $x^3 - 5x^2 + 9x - 9$? Is there a way we could find the other factor? ***Note:*** If you want a hint, skip ahead to part **c**. But try without the hint first.

c. The graph of $y_1 = x^3 - 5x^2 + 9x - 9$ has an x-intercept at the point $(3,0)$. Therefore, $x - 3$ is a factor of $x^3 - 5x^2 + 9x - 9$. This means that

$$x^3 - 5x^2 + 9x - 9 = (x - 3)(\text{some other factor})$$

Find the other factor. ***Hint:*** Long division.

d. To find the other factor, we do the long division $\dfrac{x^3 - 5x^2 + 9x - 9}{x - 3}$. Thus,

$x^3 - 5x^2 + 9x - 9 = (x - 3)(x^2 - 2x + 3)$.

Optional Challenge Problem: Can we factor this polynomial any further? Why or why not? ***Hint:*** Think about the graph.

4. Factor the polynomial $x^3 - x^2 - 12x + 32$. Use the method that you developed in the last problem. Your final answer should be the product of a linear factor and a quadratic factor. ***Hint:*** Start by graphing on your calculator. Include the sketch of the graph as part of your solution.

Answer: $(x+4)(x^2 - 5x + 8)$

5. Consider the polynomial $x^3 - 6x^2 - 3x + 18$.

a. Graph $y_1 = x^3 - 6x^2 - 3x + 18$ on your calculator. Sketch the graph below.

b. This time, there are three x-intercepts. Which one of these x-intercepts should we use to set up our long division? Why?

c. The "best" x-intercept is $(6,0)$. (The other two x-intercepts are weird decimals.) Now use the method from the last two problems to factor $x^3 - 6x^2 - 3x + 18$. Your answer should be the product of a linear and a quadratic factor.

d. Is there another, faster way that we could have factored $x^3 - 6x^2 - 3x + 18$? ***Hint:*** Factoring by grouping.

e. In parts **c** and **d**, you should have found $x^3 - 6x^2 - 3x + 18 = (x - 6)(x^2 - 3)$. Now solve the equation $x^3 - 6x^2 - 3x + 18 = 0$. ***Hint:*** You already know the factoring.

f. The equation $x^3 - 6x^2 - 3x + 18 = 0$ had three solutions: 6 and $\pm\sqrt{3}$. What are the exact coordinates for each of the three x-intercepts on the graph $y_1 = x^3 - 6x^2 - 3x + 18$?

g. The coordinates for the three x-intercepts are $(6,0)$, $(\sqrt{3},0)$, and $(-\sqrt{3},0)$. Can we factor the polynomial $x^3 - 6x^2 - 3x + 18$ into three linear factors? Do it if you can.

Answer: g. $x^3 - 6x^2 - 3x + 18 = (x - 6)(x - \sqrt{3})(x + \sqrt{3})$

6. Our goal in this problem is to factor the polynomial $x^3 - 3x^2 - 5x + 14$.

a. Factor $x^3 - 3x^2 - 5x + 14$. ***Hint:*** Which x-intercept is best to use?

b. You should have found that $x^3 - 3x^2 - 5x + 14 = (x-2)(x^2 - x - 7)$. Now solve the equation $x^3 - 3x^2 - 5x + 14 = 0$. ***Hint:*** You already have the factoring for $x^3 - 3x^2 - 5x + 14$.

c. The equation $x^3 - 3x^2 - 5x + 14 = 0$ had three solutions: 2 and $\frac{1}{2} \pm \frac{\sqrt{29}}{2}$. (In part **d**, use the quadratic formula to find $\frac{1}{2} \pm \frac{\sqrt{29}}{2}$.) What are the exact coordinates for each of the three x-intercepts on the graph $y_1 = x^3 - 3x^2 - 5x + 14$?

d. The coordinates for the three x-intercepts are $(2,0)$, $(\frac{1}{2} + \frac{\sqrt{29}}{2}, 0)$, and $(\frac{1}{2} - \frac{\sqrt{29}}{2}, 0)$. Now factor the polynomial $y_1 = x^3 - 3x^2 - 5x + 14$ into three linear factors.

Answer: d. $x^3 - 3x^2 - 5x + 14 = (x-2)\left(x - (\frac{1}{2} + \frac{\sqrt{29}}{2})\right)\left(x - (\frac{1}{2} - \frac{\sqrt{29}}{2})\right)$

7. In this problem, we will explore polynomials that are a difference of two cubes.

a. Suppose you were about to graph $y = x^3 - 8$ on your calculator. Before you graph, can you determine one of the x-intercepts? Explain.

b. We know that $(2,0)$ is an x-intercept because $2^3 - 8 = 0$. What must be a factor of $x^3 - 8$?

c. Since $(2,0)$ is an x-intercept, we can conclude that $x - 2$ is a factor of $x^3 - 8$. Do long division to find the other factor.

d. You should have found that $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$. Note that this matches the pattern for factoring a difference of cubes that we developed in a previous problem set. Now factor $x^3 - 27$. ***Note:*** You may do this either by using the pattern or by doing a long division.

8. In this problem, we will explore polynomials that are the sum of two cubes.

a. Suppose you were about to graph $y = x^3 + 125$ on your calculator. Before you graph, can you determine one of the x-intercepts? Explain.

b. We know that $(-5,0)$ is an x-intercept because $(-5)^3 + 125 = 0$. What must be a factor of $x^3 + 125$?

c. Since $(-5,0)$ is an x-intercept, we can conclude that $x + 5$ is a factor of $x^3 + 125$. Do long division to find the other factor.

d. You should have found that $x^3 + 125 = (x+5)(x^2 - 5x + 25)$. Note that this matches the pattern for factoring a sum of cubes that we developed in a previous problem set. Now factor $x^3 + 27$. ***Note:*** You may do this either by using the pattern or by doing a long division.