Algebra 2/Pre-Calculus

Name__________________

Using the Properties of Logarithms (Day 4, Logarithmic Functions)

In our last problem set, we began exploring the connection between the rules of exponents and the rules of logarithms. In this problem set, we will explore how to use (and how to think about) these rules.

Here's a table relating the rules of exponents with the rules of logarithms:

Exponent Rules		***Log Rules***
$b^x \cdot b^y = b^{x+y}$	$\longleftrightarrow$	$\log_b MN = \log_b M + \log_b N$
$\dfrac{b^y}{b^x} = b^{x-y}$	$\longleftrightarrow$	$\log_b \frac{M}{N} = \log_b M - \log_b N$
$(b^x)^a = b^{ax}$	$\longleftrightarrow$	$\log_b M^a = a \log_b M$

1. Suppose another student asked you to explain the rule $\log_b MN = \log_b M + \log_b N$. What would you say? ***Hint:*** Explain how this is related to the exponent rule $b^x \cdot b^y = b^{x+y}$.

2. Condense each of the following to a single logarithm. If possible, find the value of the logarithm. No calculators! ***Note:*** There are answers at the end of this problem.

a. $\log_7 3 + \log_7 10$

b. $\log_5 24 - \log_5 8$

c. $\log_6 3 + \log_6 12$

d. $\log 7000 - \log 7$

e. $3\log_7 5$

f. $3\log_8 2$

g. $2\log_5 3 + \log_5 4$

h. $2\log_2 6 - 2\log_2 3$

i. $2\log 50 + \log 4$

j. $\frac{1}{2}\log_6 2 + \frac{1}{2}\log_6 18$

k. $8\log_2 \sqrt{2}$

l. $\log_{30} 3 + \log_{30} 6 + \log_{30} 50$

m. $\log_2 32 + \log_5 (\frac{1}{25})$

n. $\log_9 3 + \log_7 \sqrt{7} - \log_{11} 1$

Answers a. $\log_7 30$ b. $\log_7 3$ c. 2 d. 3 e. $\log_7 125$ f. 1 g. $\log_5 36$ h. 2 i. 4 j. 1 k. 4 l. 2 m. 3 n. 1

3. Condense each of the following to a single logarithm. No calculators! ***Note:*** There are answers at the end of this problem.

a. $\log_4 A + \log_4 B$

b. $3\log_b M + 7\log_b N$

c. $\log_3 A - \log_3 B - \log_3 C$

d. $\log_b L - 2(\log_b M - \log_b N)$

Answers a. $\log_4 AB$ b. $\log_b(M^3N^7)$ c. $\log_3\left(\frac{A}{BC}\right)$ d. $\log_b\left(\frac{LN^2}{M^2}\right)$

4. Expand each of the following. ***Note:*** There are answers at the end of this problem.

a. $\log_6(5A)$

b. $\log_7\left(\frac{XY}{Z}\right)$

c. $\log(A^3\sqrt{B})$

b. $\log_5\left(\frac{\sqrt{M}}{N^4}\right)$

Answers a. $\log_6 5 + \log_6 A$ b. $\log_7 X + \log_7 Y - \log_7 Z$ c. $3\log A + \frac{1}{2}\log B$

d. $\frac{1}{2}\log_5 M - 4\log_5 N$

5. What numbers can we take the log of? Can we take the log of positive numbers? Negative numbers? Zero? ***Note:*** We'll use this idea in the next problem.

6. We can only take logs of positive numbers. (Make sure you understand why this is true.)

a. Solve the equation: $\log_2(x) + \log_2(x+6) = 4$. ***Hint:*** Use the exponent rules, then solve a quadratic equation.

b. Here's a student's solution for the problem from part **a**:

$$\log_2(x) + \log_2(x+6) = 4$$

$$\log_2(x^2 + 6x) = 4$$

$$x^2 + 6x = 2^4$$

$$x^2 + 6x = 16$$

$$x^2 + 6x - 16 = 0$$

$$(x+8)(x-2) = 0$$

$$x = -8 \text{ or } x = 2$$

Do you agree with this student's solution? Explain why or why not. ***Hint:*** Think about problem **5**.

7. In the last problem, the student solution is almost correct, but $x = -8$ doesn't work because it would result in taking a log of a negative number. (We call this an *extraneous solution*.) Hence, the only true solution is $x = 2$.

Let's take another look at the last equation: $\log_2(x) + \log_2(x+6) = 4$. Explain how you can tell immediately that any solution to this equation must satisfy the following inequalities: $x > 0$ and $x > -6$.

8. Solve each of the following equations. ***Caution:*** Some (but not all) of these equations have extraneous solutions. Hint: Start by writing inequalities for any restrictions on *x*.

a. $\log_2(x-3) + \log_2(x+1) = 5$

b. $\log_3(x-5) + \log_3(x+3) = 2$

c. $\log_8 2 + \log_8(x^2) = 1$

d. $\log_2(x^2 + 4x + 3) = 3$

e. $\log 5 + \log(x+8) = 2$

f. $\log_2(x+10) - \log_2 5 = 3$

g. $2\log_6 x + \log_6 9 = 2$

h. $\log_6 x^2 = 2 - \log_6 3$

i. $\log_2 x + \log_2(x^2 + 4x - 1) = 2$

j. $\log(3 - 4x) = \log(x - 22)$

Answer a. 7 only b. 6 only c. ± 2 (both work) d. 1 or -5 (both work) e. 12 only f. 30 g. 2 only h. $\pm\sqrt{12}$ (both work) i. 1 only j. No solution

8. Andrew and Lucas were trying to solve the equation $2\log_3 x = 4$. Their work is shown below.

Andrew's Work	***Lucas's Work***
$2\log_3 x = 4$	$2\log_3 x = 4$
$\log_3 x^2 = 4$	$\log_3 x = 2$
$x^2 = 3^4$	$x = 3^2$
$x^2 = 81$	$x = 9$
$x = \pm 9$	

Did they both do the problem correctly? Does one of them have a better approach? Explain.

9. Alison and Jessie were trying to solve the equation $3\log_2(3x+7)=12$, but they started the problem in different ways:

Alison's Start	***Jessie's Start***
$3\log_2(3x+7)=12$	$3\log_2(3x+7)=12$
$\log_2(3x+7)^3=12$	$\log_2(3x+7)=4$

a. Whose approach is better? Why?

b. Finish solving the equation.

10. Consider the equation: $\log_7(2x+3)=\log_7 11$.

a. Solve the equation.

b. Here are two different solutions to the equation.

Solution 1

$\log_7(2x+3) = \log_7 11$

$\log_7(2x+3) - \log_7 11 = 0$

$\log_7\left(\frac{2x+3}{11}\right) = 0$

$\frac{2x+3}{11} = 7^0$

$\frac{2x+3}{11} = 1$

$2x+3 = 11$

$2x = 8$

$x = 4$

Solution 2

$\log_7(2x+3) = \log_7 11$

$2x+3 = 11$

$2x = 8$

$x = 4$

What are the advantages and disadvantages to each solution? Explain.

11. Solve each of the following equation. ***Caution:*** Some of these equations might have extraneous solutions.

a. $\log_8 25 = \log_8(3x+7)$

b. $\log_5(3x+1) = \log_5(13-x)$

c. $\log_7(7-2x) = \log_7(x-8)$

d. $\log(6x+30) = \log(x+10)$

Answers a. 6 b. 3 c. no solution (5 is extraneous) d. -4 (not extraneous)