Algebra 2/Pre-Calculus **Name**_____

Adding and Subtracting (Day 2, Rational Expressions) (DDMath.com)

In this problem set, we will apply the method of adding and subtracting rational numbers to rational expressions.

1. Add the following fractions.

a. $\frac{1}{3}+\frac{2}{5}$

b. $\frac{5}{6}-\frac{4}{9}$

2. You should have found that $\frac{1}{3}+\frac{2}{5}=\frac{5}{15}+\frac{6}{15}=\frac{11}{15}$ and $\frac{5}{6}-\frac{4}{9}=\frac{15}{18}-\frac{8}{18}=\frac{7}{18}$. Notice that in each of these computations, we had to find a common denominator before adding or subtracting. In this problem, we will do the same type of computations with rational expressions.

a. Simplify: $\frac{4}{3x}+\frac{2}{x^2}$.

b. You should have found that $\frac{4}{3x}+\frac{2}{x^2}=\frac{4x}{3x^2}+\frac{6}{3x^2}=\frac{4x+6}{3x^2}$. Now simplify $\frac{3}{x+5}-\frac{2}{x+4}$.

3. Here's what you should have found on the last problem:

$$\frac{3}{x+5}-\frac{2}{x+4}=\frac{3(x+4)}{(x+5)(x+4)}-\frac{2(x+5)}{(x+5)(x+4)}=\frac{3x+12}{(x+5)(x+4)}-\frac{2x+10}{(x+5)(x+4)}=\frac{x+2}{(x+5)(x+4)}$$

Use this same type of approach simplify each of the following expressions. ***Note:*** Answers are provided at the end of this problem.

a. $\frac{5}{3x}-\frac{2}{3x}$

b. $\frac{6}{5x}-\frac{3}{x}$

c. $\frac{x+1}{x^2}-\frac{2}{x}$

d. $\frac{2}{x-3}+\frac{3}{x-2}$

e. $\dfrac{x+1}{x^2-4}-\dfrac{3}{x+2}$

f. $\dfrac{x-1}{x}-\dfrac{x}{x-1}$

g. $\dfrac{3x}{x^2-9}+\dfrac{4}{x^2+3x}$

Answers a. $\dfrac{1}{x}$ b. $\dfrac{-9}{5x}$ c. $\dfrac{-x+1}{x^2}$ d. $\dfrac{5x-13}{(x-2)(x-3)}$ e. $\dfrac{-2x+7}{x^2-4}$ f. $\dfrac{-2x+1}{x(x-1)}$

g. $\dfrac{3x^2+4x-12}{x(x+3)(x-3)}$

4. Consider the following problem: Simplify $\frac{x+3}{x+6}-\frac{x-2}{x+6}$.

a. Simplify: $\frac{x+3}{x+6}-\frac{x-2}{x+6}$.

b. James and Sasha were both working on this problem. Here is the work that each of them did.

James's Work

$$\frac{x+3}{x+6}-\frac{x-2}{x+6}$$
$$=\frac{x+3-x-2}{x+6}$$
$$=\frac{1}{x+6}$$

Sasha's Work

$$\frac{x+3}{x+6}-\frac{x-2}{x+6}$$
$$=\frac{x+3-(x-2)}{x+6}$$
$$=\frac{x+3-x+2}{x+6}$$
$$=\frac{5}{x+6}$$

Who was right? Explain.

c. Suppose $f(x)=\dfrac{x+3}{x+6}-\dfrac{x-2}{x+6}$, $g(x)=\dfrac{1}{x+6}$, and $h(x)=\dfrac{5}{x+6}$. Find $f(5)$, $g(5)$, and $h(5)$. How does this relate to the question about James and Sasha?

d. You should have found the following:

$$f(5)=\frac{5+3}{5+6}-\frac{5-2}{5+6}=\frac{8}{11}-\frac{3}{11}=\frac{5}{11},\ g(5)=\frac{1}{5+6}=\frac{1}{11},\ h(5)=\frac{5}{5+6}=\frac{5}{11}$$

Explain why this demonstrates that James had the wrong answer.

e. Simplify $\dfrac{4x+7}{x-2}-\dfrac{2x+4}{x-2}$.

Answer e. $\dfrac{2x+3}{x-2}$

5. Simplify each of the following expressions. ***Note:*** Answers are provided at the end of this problem.

a. $\dfrac{x+1}{x-4}-\dfrac{x+2}{x+6}$

b. $\dfrac{2x-1}{x-3}-\dfrac{4x+2}{x+3}$

c. $\dfrac{x}{x-3}-\dfrac{x-2}{x^2-2x-3}$

d. $\dfrac{x-1}{x^3-4x}-\dfrac{3}{x^3-2x^2}$

Answers a. $\dfrac{9x+14}{(x-4)(x+6)}$ b. $\dfrac{-2x^2+15x+3}{(x-3)(x+3)}$ c. $\dfrac{x^2+2}{(x-3)(x+1)}$ d. $\dfrac{x^2-4x-6}{x^2(x+2)(x-2)}$

6. More problems! Simplify.

a. $\dfrac{x^3 - 7x^2 + 12x}{x^2 - 16}$

b. $\dfrac{5}{x^2} \cdot \dfrac{2x}{35}$

c. $\left(\dfrac{1}{x-4}\right)^2$

d. $\dfrac{x^3 - 8}{x^2 + 2x + 4}$

e. $\left(\dfrac{2x}{3}\right)^2$

f. $\left(\dfrac{2x}{3}\right)^3$

g. $\dfrac{x^2}{x+3}+\dfrac{7x+12}{x+3}$

h. $\dfrac{x^3-15}{x^2+5}-\dfrac{3x^2-5x}{x^2+5}$

i. $(x+5)\left(\dfrac{x-2}{x+2}+\dfrac{x-4}{x+5}\right)$

j. $\left(\dfrac{x-3}{x-2}\right)\left(\dfrac{x-2}{x+7}+\dfrac{x-2}{x+4}\right)$

Answers a. $\dfrac{x^2-3x}{x+4}$ b. $\dfrac{2}{7x}$ c. $\dfrac{1}{(x-4)^2}=\dfrac{1}{x^2-8x+16}$ d. $x-2$ e. $\dfrac{4x^2}{9}$ f. $\dfrac{8x^3}{27}$

g. $x+4$ h. $x-3$ i. $\dfrac{2x^2+x-18}{x+2}$ j. $\dfrac{2x^2+5x-33}{(x+7)(x+4)}$