MATH 3 **Name**_____

Division and More Practice (Day 4, Rational Expressions) (DDMath.com)

In this problem set, we will learn how to divide fractions involving variables. As you do this assignment, think back to the knowledge of fractions that you already have.

1. In this problem, we will begin exploring fractions within fractions.

a. Simplify the following fraction: $\dfrac{3}{\frac{2}{5}}$.

b. Here's a way of simplifying the last fraction: $\dfrac{3}{\frac{2}{5}} = \dfrac{3}{\frac{2}{5}} \cdot \dfrac{5}{5} = \dfrac{15}{2}$. Notice that the key step is multiplying the top and bottom of the fraction by the same amount. Now use a similar approach to simplify the fraction $\dfrac{\frac{2}{3}}{7}$.

c. You can simplify the fraction in the following way: $\dfrac{\frac{2}{3}}{7} = \dfrac{\frac{2}{3}}{7} \cdot \dfrac{3}{3} = \dfrac{2}{21}$. Now simplify each of the following fractions.

i. $\dfrac{\frac{1}{2}}{8}$ **ii.** $\dfrac{8}{\frac{2}{7}}$

Answers i. $\dfrac{1}{16}$ ii. 28

2. In this problem, we will explore what happens when we divide a fraction by a fraction.

a. Simplify the following fraction: $\dfrac{\frac{3}{5}}{\frac{2}{5}}$.

b. Here's a way to simplify it: $\dfrac{\frac{3}{5}}{\frac{2}{5}} \cdot \dfrac{5}{5} = \dfrac{3}{2}$. Use a similar strategy to simplify $\dfrac{\frac{3}{8}}{\frac{7}{8}}$.

c. You should have found that $\dfrac{\frac{3}{8}}{\frac{7}{8}} = \dfrac{3}{7}$. Now simplify $\dfrac{\frac{3}{5}}{\frac{2}{3}}$.

d. Here's a way to simplify it: $\dfrac{\frac{3}{5}}{\frac{2}{3}} \cdot \dfrac{15}{15} = \dfrac{\frac{45}{5}}{\frac{30}{3}} = \dfrac{9}{10}$. Now simplify $\dfrac{\frac{1}{2}}{\frac{3}{7}}$.

Answer d. $\dfrac{7}{6}$

3. In this problem, another way of dividing by a fraction.

a. Find the value of $\dfrac{\frac{4}{5}}{\frac{7}{9}}$.

b. You should have found that $\dfrac{\frac{4}{5}}{\frac{7}{9}} = \dfrac{36}{35}$. Now find the value of $\dfrac{4}{5} \cdot \dfrac{9}{7}$.

c. You should have found that $\dfrac{4}{5} \cdot \dfrac{9}{7} = \dfrac{36}{35}$. Notice that this is the same value we found for $\dfrac{\frac{4}{5}}{\frac{7}{9}}$. Why does this happen?

d. Simplify $\dfrac{\frac{a}{b}}{\frac{c}{d}}$ and $\dfrac{a}{b} \cdot \dfrac{c}{d}$.

e. You should have found that $\dfrac{\frac{a}{b}}{\frac{c}{d}} = \dfrac{\frac{a}{b}}{\frac{c}{d}} \cdot \dfrac{\frac{d}{c}}{\frac{d}{c}} = \dfrac{\frac{ad}{bc}}{\frac{1}{1}} = \dfrac{ad}{bc}$ and $\dfrac{a}{b} \cdot \dfrac{c}{d}$.
Explain how these calculations are related to the idea developed earlier in this problem.

4. In the last problem, we observed that dividing by a fraction is the same as multiplying by the reciprocal of that fraction. Use this idea to simplify each of the following.

a. $\dfrac{\frac{1}{3}}{\frac{7}{10}}$

b. $\dfrac{\frac{2}{7}}{\frac{4}{5}}$

c. $\frac{3}{4} \div \frac{2}{5}$

d. $\frac{2}{5} \div \frac{8}{15}$

Answers a. $\frac{10}{21}$ b. $\frac{5}{14}$ c. $\frac{15}{8}$ d. $\frac{3}{4}$

5. We will now begin simplifying fractions that involve variables. Simplify each of the following.

a. $\dfrac{\frac{x}{3}}{\frac{4}{x}}$

b. $\frac{2x^3}{3y} \div \frac{6x^2}{y}$

c. $\dfrac{x^2 - x - 12}{x^2 - 25} \div \dfrac{x - 4}{x - 5}$

Solutions a. $\dfrac{\frac{x}{3}}{\frac{4}{x}} = \dfrac{x}{3} \cdot \dfrac{x}{4} = \dfrac{x^2}{12}$ b. $\dfrac{2x^3}{3y} \div \dfrac{6x^2}{y} = \dfrac{2x^3}{3y} \cdot \dfrac{y}{6x^2} = \dfrac{x}{9}$

b. $\dfrac{x^2 - x - 12}{x^2 - 25} \div \dfrac{x - 4}{x - 5} = \dfrac{(x+3)(x-4)}{(x+5)(x-5)} \cdot \dfrac{x-5}{x-4} = \dfrac{x+3}{x+5}$

6. In this problem, we will explore fractions within fractions.

a. Simplify the fraction $\dfrac{\frac{3}{4}}{3 + \frac{1}{2}}$. ***Hint:*** Start by finding a common denominator for the fractions on the bottom.

b. Here's how you could have approached the last problem:

$$\frac{\frac{3}{4}}{3+\frac{1}{2}} = \frac{\frac{3}{4}}{\frac{6}{2}+\frac{1}{2}} = \frac{\frac{3}{4}}{\frac{7}{2}} = \frac{3}{4}\cdot\frac{2}{7} = \frac{3}{14}$$

Now use the same type of strategy to simplify $\dfrac{\frac{x}{2}+3}{\frac{3}{x}}$.

Solution: $\dfrac{\frac{x}{2}+3}{\frac{3}{x}} = \dfrac{\frac{x}{2}+\frac{6}{2}}{\frac{3}{x}} = \dfrac{\frac{x+6}{2}}{\frac{3}{x}} = \dfrac{x+6}{2}\cdot\dfrac{x}{3} = \dfrac{x^2+6x}{6}$

7. Simplify the following expressions:

a. $\dfrac{x^2-5x+6}{x^2-4}\cdot\dfrac{2x+4}{2x^2-18}$

b. $\dfrac{x^3+8}{2x^2-2}\cdot\dfrac{2x^2+4x+2}{x^2+3x+2}$

c. $\dfrac{x^4-4x^2-45}{x^3+3x^2} \div \dfrac{2x^3+10x}{4x}$

d. $\dfrac{1-x^2}{3x} \cdot \dfrac{x^2}{x^2+x}$

e. $\dfrac{x^3-2x^2+4x-8}{2x-4} \cdot \left(x^2+4\right)^{-1}$

f. $\dfrac{2x^2-2x}{2x^2+4x-6} \cdot \left(2x^2-18\right)$

g. $\dfrac{4x^2-25}{3x}\cdot\left(x^3-x\right)\div\dfrac{4x+10}{6x+6}$

Answers a. $\dfrac{1}{x+3}$ b. $\dfrac{x^2-2x+4}{x-1}$ c. $\dfrac{2(x-3)}{x^2}$ d. $\dfrac{1-x}{3}$ e. $\dfrac{1}{2}$ f. $2x(x-3)$ g. $(2x-5)(x+1)^2(x-1)$

8. Simplify the following expressions:

a. $\dfrac{3}{x-2}+\dfrac{x}{x+4}$

b. $\dfrac{x-2}{x+3}-\dfrac{1}{2}$

c. $\dfrac{x+4}{x^2-4}-\dfrac{3}{x+2}$

d. $\dfrac{x+5}{x^2-2x-3}-\dfrac{x-1}{x^2-3x}$

e. $\dfrac{5}{3x^2}-\dfrac{4x}{x+1}$

f. $\dfrac{2}{3}+\dfrac{4}{x}-\dfrac{x}{x+3}$

Answers a. $\dfrac{x^2+x+12}{(x-2)(x+4)}$ b. $\dfrac{x-7}{2(x+3)}$ c. $\dfrac{-2x+10}{(x+2)(x-2)}$ d. $\dfrac{5x+1}{x(x-3)(x+1)}$

e. $\dfrac{-12x^3+5x+5}{3x^2(x+1)}$ f. $\dfrac{-x^2+18x+36}{3x(x+3)}$

9. Simplify the following expressions:

a. $\dfrac{\dfrac{3}{x}+x}{3-\dfrac{4}{x}}$

b. $\dfrac{\dfrac{1}{x-1}+5}{x+\dfrac{2}{x-1}}$

Answers a. $\dfrac{x^2+3}{3x-4}$ b. $\dfrac{5x-4}{x^2-x+2}$

10. Solve the following equations

a. $\dfrac{3}{x}+\dfrac{1}{4}=\dfrac{5}{2x}$

b. $\dfrac{2}{x-5}+1=7$

c. $\frac{x}{x-2}+3=\frac{2}{x-2}$

d. $\frac{8}{x+6}=\frac{x}{2x-2}$

e. $\frac{1}{x+2}+\frac{2x}{x-1}=2$

f. $\frac{x+5}{x^2-1}=\frac{x-3}{x+1}$

g. $\frac{4x+1}{x+1} = \frac{12}{x^2-1} + 3$

h. $\frac{12}{x+1} - \frac{3}{x-2} = 1$

Answers a. -2 b. 16/3 c. no solution (x=2 is extraneous) d. 8, 2 e. -1
f. $\frac{5 \pm \sqrt{33}}{2}$ g. 5, -2 h. 5