Algebra 2/Pre-Calculus

Name__________________

Expected Value (Day 2, Statistics)

In this handout, we will continue exploring the concept of expected value.

1. Suppose you role a standard 6-sided dice. What is the average value for one role of this die?

Answer $\frac{1}{6}(1+2+3+4+5+6)=\frac{1}{6}(1)+\frac{1}{6}(2)+\frac{1}{6}(3)+\frac{1}{6}(4)+\frac{1}{6}(5)+\frac{1}{6}(6)=3.5$

2. Suppose instead you had a 10 sided die with the following numbers {3, 3, 3, 3, 7, 7, 7, 7, 7, 7}.

a. What is the probability of rolling a 3?

b. What is the probability of rolling a 7?

c. What is the average value for one roll of this die?

Answer a. $\frac{4}{10}$ b. $\frac{6}{10}$ c. $\frac{1}{10}(4(3)+6(7))=\frac{4}{10}(3)+\frac{6}{10}(7)=5.4$

3. Notice that in the last two problems, we found the average value by taking the sum of each outcome multiplied by its probability. Now suppose we have a six sided die with the numbers 1, 2, 3, 4, 5, and 6. This time, the die is unfair. 50% of the time, it lands on 3. It lands on each of the other numbers 10% of the time.

a. Would the average value for this die be higher or lower than a standard six sided die? Explain.

b. Find average value for one roll of this die. ***Hint:*** Multiply each outcome by its probability and add up the values you get.

Answer a. Lower. (But why?) b. $0.1(1)+0.1(2)+0.5(3)+0.1(4)+0.1(5)+0.1(6)=3.3$

Expected Value

In the last few problems, you were asked to find the "average value." Mathematicians refer to this as the ***expected value***, which is calculated in the following way. First, multiply the value for each outcome by the probability of that outcome. Then take the sum. We will practice finding expected value in the next few problems.

4. At a certain high school, there are lots of announcements on the loudspeaker. Suppose there's a 30% chance of having 0 announcements, a 30% chance of having 1 announcement, a 20% chance of having 2 announcements, a 10% chance of having 3 announcements, and a 10% chance of having 4 announcements. Find the *expected value* of the number of announcements per day.

Answer $0.3(0)+0.3(1)+0.2(2)+0.1(3)+0.1(4)=1.4$

5. A snack machine is malfunctioning. Sometimes when you put the cost of a snack in the machine, it gives you more than 1 snack, or it gives you no snack at all. Here are the probabilities.

x = how many snacks you get	probability
0	0.04
1	0.92
2	0.03
3	0.01

a. Calculate the *expected value* for how many snacks you get.

b. On average, is the machine giving out more snacks than it should, or less snacks than it should? Explain how you know.

Answers a. $0.04(0)+0.92(1)+0.03(2)+0.01(3)=1.01$ b. On average, the machine gives out slightly more than it is supposed to. (The expected value is greater than 1.)

6. Suppose a coin is flipped 4 times, and let x equal the number of times that the coin is heads.

x = # of heads	probability
0	
1	
2	
3	
4	

a. Fill in the missing probabilities in the table.

Hint: These probabilities are related to Pascal's Triangle and the combination numbers. Do you remember how?

b. Calculate the *expected value* for the number of heads.

c. Now suppose a coin is flipped 5 times. Calculate the expected value for the number of heads. ***Suggestion:*** Start by making a table similar to the one in the previous problem.

7. Suppose you roll a six sided die and an eight sided die with the following numbers on each side: $\{5, 5, 7, 8, 8, 8\}$ and $\{2, 2, 7, 7, 7, 7, 9, 9\}$.

a. Write out the associated polynomial multiplication. Then do it. You may use wolframalpha, if you want.

b. What is the expected value for the sum?

8. Look at the "Plinko" game on page 195 in the textbook. (The triangle with the dots that says, "DROP CHIP HERE."

a. How many total paths are there that the chip can take as it falls?

b. You should have found that there were 256 possible paths. (There are two possible ways it can go each time and $2^8 = 256$.) Suppose you want the chip to get to the 1000 on the left. To get to this spot, how many times does the chip need to go left and how many times does it need to go right?

c. You should have found that the chip needed to go left six times and right twice. For example, if the chip went LRLLLRLL, it would land on that 1000. It would also get to that spot if it went LLRRLLLL. (Trace this out with your finger to make sure you see it.) How many ways are there for the chip to land on the 1000? ***Big hint:*** There are 8 moves and you are choosing 2 of them to be right.

d. You should have found that there were ${}_8C_2 = 28$ ways of hitting this spot. What is the probability of hitting this spot?

e. Make a table giving the probability of each outcome. Your table will be similar to the ones you made in problem 3.

f. Find the average amount a player will win, per chip, in the long run. ***Note:*** This is problem 3 on page 195, so you can check your answer in the back of the book.

More Homework Problems!!!!

From the textbook, page 195-6: 4, 6-10

Don't "forget" to do the book problems!