Algebra 2/Pre-Calculus

Name__________________

Geometric and Inverse Trigonometry

In this handout, we will practice using the triangle area formula, the Law of Sines, and the Law of Cosines in a variety of practice problems.

Important: Draw a picture for each problem. Try to make your drawing reasonably accurate. (Acute angles should look acute and obtuse angles should look obtuse. The sides of the triangle that are longer should actually look longer. Same with the angles.)

1. Prove that the area of ΔABC is given by $\frac{1}{2}ab\sin C$.

2. Prove the Law of Sines.

3. Prove the Law of Cosines.

4. Solve each of the following equations for $0^\circ \leq \theta \leq 360^\circ$.

  a. $\cos\theta = 0.32$

  b. $5\sin\theta + 4 = 2$

  c. $7\sin^2\theta = 2\sin\theta$

  d. $3\sin^2\theta = 5\cos\theta + 5$

5. In ΔABC, $\angle A = 37^\circ$, $AB = 5$, and $AC = 8$. Find the area of ΔABC.

6. In ΔDEF, $\angle F = 34^\circ$, $DE = 10$, and $EF = 11$. Is the area of ΔDEF equal to $\frac{1}{2}(10)(11)\sin 34$? Explain.

7. In ΔXYZ, $\angle Y = 70^\circ$, $\angle Z = 25^\circ$, and $YZ = 13$. Find all missing sides and angles for ΔXYZ.

8. In ΔABC, $a = 20$, $b = 12$, and $c = 9$.

  a. Find all missing sides and angles for ΔABC.

  b. Find the area of ΔABC.

9. In ΔXYZ, $x = 10$, $z = 14$, and $\angle X = 25^\circ$. How many different ways can this triangle be drawn? Draw all possibilities in the space below. ***Note:*** When we say “different” in this context, we mean “not congruent.” Drawing exactly the same triangle in a different spot doesn’t count.

10. You should have found that there were two ways of drawing ΔXYZ from problem **9**. Find all missing angles and sides for both possible triangles.

11. In ΔABC, $AB = 14$, $AC = 17$, and $\angle C = 62^\circ$. How many different ways can this triangle be drawn? Draw all possibilities in the space below.

12. In the last problem, you should have found that ΔABC was impossible. (No such triangle can be formed.) Make sure you see why this is true. Now consider ΔDEF in which $d = 9$, $f = 18$, and $\angle D = 30^\circ$. How many different ways can this triangle be drawn? Draw all possibilities in the space below.

13. In the last problem, you should have found that ΔDEF could only be formed in exactly one way. What special type of triangle was ΔDEF? How do you know?

14. In the last problem, you should have found that ΔDEF had to be a right triangle. Without using your calculator, find the exact value of side DF. ***Hint:*** Think about the angles in ΔDEF.

15. In quadrilateral ABCD, $AB = 16$, $BC = 15$, $CD = 10$, $AD = 14$, and $\angle C = 110^\circ$. Find the area of the quadrilateral. ***Hint:*** Start by drawing diagonal BD.

Some Answers (Not Yet Double Checked!!)

4a. 71.34 or 288.66 b. 203.58 or 336.42 c. 0, 16.60, 163.40, or 180 d. 131.81, 180, 228.19

5. 12.04 6. No, 34° is not the included angle. 7. $\angle X = 85^\circ$, $y = 12.26$, $z = 5.52$

8a. $\angle A = 144.11^\circ$, $\angle B = 20.59^\circ$, $\angle C = 15.30^\circ$ b. 31.66 9. Two different ways

10. Case 1: $\angle Z = 36.28^\circ$, $\angle Y = 118.72^\circ$, $y = 20.75$ OR Case 2: $\angle Z = 143.72^\circ$, $\angle Y = 11.28^\circ$, $y = 4.63$ 11. Impossible! Side YZ is not long enough to reach the other side of the triangle.

12. Only one triangle formed. 13. It is a right triangle. 14. $9\sqrt{3}$ 15. 182.31