Algebra 2/Pre-Calculus **Name**_____

Harder Problems (Day 5, Rational Expressions) (DDMath.com)

In this problem set, we will further explore the connections between polynomials and rational expressions.

1. Consider the following rational expression: $\dfrac{x^3 - x^2 + 3x - 10}{x^3 + 4x^2 + 8x + 15}$.

a. Simplify the expression. ***Note:*** There are hints in the later part of the problem. But try it on your own first!

b. Can you factor $x^3 - x^2 + 3x - 10$? ***Hint:*** Start by graphing it on your calculator.

c. You should have found that $x-2$ was a factor of $x^3-x^2+3x-10$. What is the other factor of $x^3-x^2+3x-10$? ***Hint:*** Long division.

d. You should have found that $x^3-x^2+3x-10=(x-2)(x^2+x+5)$. Now factor $x^3+4x^2+8x+15$.

e. Simplify $\dfrac{x^3-x^2+3x-10}{x^3+4x^2+8x+15}$.

Answer $\dfrac{x^3-x^2+3x-10}{x^3+4x^2+8x+15}=\dfrac{(x-2)(x^2+x+5)}{(x+3)(x^2+x+5)}=\dfrac{x-2}{x+3}$

2. Simplify the following rational expression: $\dfrac{x^3+3x^2-3x+35}{x^3-3x^2+9x-7}$.

Answer $\dfrac{x^3+3x^2-3x+35}{x^3-3x^2+9x-7}=\dfrac{(x+5)(x^2-2x+7)}{(x-1)(x^2-2x+7)}=\dfrac{x+5}{x-1}$

3. Suppose $f(x)$ is a cubic polynomial such that $f(1) = f(-2) = f(-3) = 0$ and $f(2) = 40$.

a. Find the formula for $f(x)$. Leave your answer in factored form.

b. You should have found that $f(x) = 2(x-1)(x+2)(x+3)$. Suppose $g(x)$ is another cubic polynomial such that $g(0) = g(-2) = g(-3) = 0$ and $g(1) = 36$. Find the formula for $g(x)$.

c. You should have found that $g(x) = 3x(x+2)(x+3)$. Now simplify the rational expression $\dfrac{f(x)}{g(x)}$.

Answer $\dfrac{f(x)}{g(x)} = \dfrac{2(x-1)(x+2)(x+3)}{3x(x+2)(x+3)} = \dfrac{2x-2}{3x}$

4. Suppose $f(x)$ is a cubic polynomial such that $f(-2) = f(2) = f(5) = 0$ and $f(0) = 10$. Suppose further that $g(x)$ is a quadratic polynomial such that $g(-2) = g(5) = 0$ and $g(1) = -8$. Simplify the rational expression $\dfrac{f(x)}{g(x)}$.

Answer $\dfrac{f(x)}{g(x)} = \dfrac{\frac{1}{2}(x+2)(x-2)(x-5)}{\frac{2}{3}(x+2)(x-5)} = \dfrac{\frac{1}{2}(x-2)}{\frac{2}{3}} = \dfrac{3(x-2)}{4} = \dfrac{3x-6}{4}$

5. Consider the rational expression $\frac{x^3-3x^2+5x-15}{x^2+x-12}$.

a. Perform the long division $\frac{x^3-3x^2+5x-15}{x^2+x-12}$.

b. You should have found that $\frac{x^3-3x^2+5x-15}{x^2+x-12}=x-4+\frac{21x-63}{x^2+x-12}$. Simplify the rational expression $\frac{x^3-3x^2+5x-15}{x^2+x-12}$ by factoring the numerator and the denominator.

c. You should have found that $\frac{x^3-3x^2+5x-15}{x^2+x-12}=\frac{x^2+5}{x+4}$. Now perform the long division $\frac{x^2+5}{x+4}$.

d. You should have found that $\frac{x^2+5}{x+4}=x-4+\frac{21}{x+4}$. Does this match the answer you got in part **a**? Explain.