Algebra 2/Pre-Calculus

Name_________________

Introducing Mean, Variance, and Standard Deviation (Day 3, Statistics)

In this handout, we will begin to explore the concepts of mean, deviation, variance and standard deviation.

1. Find the mean of the following set of numbers: $\{2, 7, 8, 8, 9\}$. ***Note:*** The mean is the average of the numbers. More specifically, the mean is the expected value if one of these numbers is selected at random.

2. You should have found that the mean in the last problem was 6.8. (Because $\frac{1}{5}(2+7+8+8+9)=6.8$). The goal of this problem is to write a formula for the mean. ***Note:*** You will have to use a Σ in part **c**.

a. $\{a,b,c,d,e\}$

b. $\{x_1,x_2,x_3,x_4,x_5\}$

c. $\{x_1,x_2,\ldots,x_n\}$

Answers a. $\frac{1}{5}(a+b+c+d+e)$ b. $\frac{1}{5}(x_1+x_2+x_3+x_4+x_5)$ c. $\frac{1}{n}(x_1+x_2+\ldots+x_n)=\frac{1}{n}\sum_{i=1}^{n}x_i$

Definition "Mean" The mean of a set of numbers, $\{x_1, x_2, ..., x_n\}$, is given by the formula $\bar{x} = \frac{1}{n}(x_1 + x_2 + ... + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i$. We use the symbol $\bar{x}$ for the mean. Note: $\bar{x}$ is sometimes called "x bar."

Important note: The calculation of a mean should feel very similar to the calculation of expected value. In this course, unless otherwise noted, we will use these concepts interchangeably.

3. You are playing a game in which you spin one of the following spinners. You get to pick whichever spinner you want. Your prize, in dollars, is whatever number you land on. (For example, if you use spinner A and get a 9, you would get $9.)

Spinner A: {6,7,8,9,10}

Spinner B: {1,3,5,12,19}

Spinner C: {8,8,8,8,8}

a. Find the expected value for each spinner. ***Note:*** This is the same as the mean for each spinner.

b. You should have found that each of these spinners had a mean of 8. And yet, we can see that they are all different. Describe in words how the spinners differ from each other.

c. Suppose you could only spin once. Which spinner would you spin and why? **Explain in words!**

Deviation and Variance

Deviation measures how far our values are from the mean. The concept of deviation is explored in the next example.

3. Suppose you have a standard six sided die.

a. Find the mean.

b. You should have found that $\bar{x} = 3.5$. (Because $\bar{x} = \frac{1}{6}(1+2=3+4+5+6) = 3.5$.)

We define the "deviation from the mean" to be how far each number is from the mean. Complete the table below giving the deviation for each of the six numbers.

x	Deviation: $x - \bar{x}$
1	
2	
3	
4	
5	
6	

c. You should have found that the deviations were as follows: -2.5, -1.5, -0.5, 0.5, 1.5. 2.5. Now find the sum of the deviations.

d. You should have found that the sum of the deviations for a standard six sided die was 0. Now suppose we have an eight sided die with the following numbers: 2, 3, 3, 5, 5, 5, 17, 32. Find the sum of the deviations for this die. ***Note:*** To do this, you will need to start by finding the mean, then find each of the deviations from the mean, and then find the sum of these deviations.

Answer d. Mean: 9. Deviations: -7, -6, -6, -4, -4, -4, 8, 23. Sum of deviations = 0.

4. Notice that in our last problem, the sum of the deviations from the mean was equal to 0 for both dice. It turns out that this will always happen. Explain why.

5. **Optional Challenge** Prove that the sum of the deviations from the mean is always equal to 0.

6. Let's return to the standard 6-sided die.

a. Since the sum of the deviations is always 0, we often consider the squared deviations instead. Fill in the last column of the table. (The table still refers to the six sided die from the beginning of the problem.) ***Note:*** You can check your answer for this problem in the second table on page 212 in the textbook.

x	Deviation: $x - \bar{x}$	Squared Deviation: $(x - \bar{x})^2$
1	-2.5	
2	-1.5	
3	-0.5	
4	0.5	
5	1.5	
6	2.5	

f. Find the sum of the squared deviations. In other words, find $\sum_{i=1}^{6}(x_i - \bar{x})^2$.

g. Find the mean of the squared deviations. In other words, find $\frac{1}{6}\sum_{i=1}^{6}(x_i - \bar{x})^2$.

Note: This is just your answer to part **f**, divided by 6.

Important note: This is called the ***variance***.

h. Find the square root of the mean of the squared deviations. In other words, find $\sqrt{\frac{1}{6}\sum_{i=1}^{6}(x_i - \bar{x})^2}$.

Note: This is just the square root of your answer to part **g**.

Important note: This is called the ***standard deviation***.

Some answers g. 2.917 h. 1.708

6. Let's return to the 8-sided die with sides of 2, 3, 3, 5, 5, 5, 17, 32. Find the mean, variance, and standard deviation for this die.

Answers Mean = 9, Variance = 95.25, Standard Deviation = 9.76

Bookwork: Read the section “Developing Habits of Mind” on page 213 and do page 215, 1-4 (parts a, c, and d only; you can skip part b for these problems).