Algebra 2/Pre-Calculus

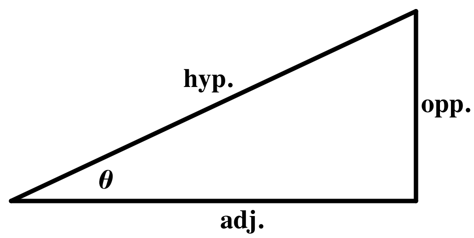
Name__________________

Introduction (Day 1, Circular Trig)

In this problem set, we will introduce a new way of defining the trigonometric functions that will apply to angles of any size (as opposed to our definitions involving the lengths of the opposite, adjacent, and hypotenuse, which can only by used for acute angles).

Definitions: Right Triangle Trigonometry

In a right triangle, we define sine, cosine, and tangent in the following way:



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

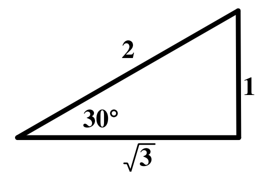
1. Draw a 30-60-90 triangle and determine the values of $\cos 30^\circ$, $\sin 30^\circ$, and $\tan 30^\circ$.

2. Explain why the definition provided above is insufficient for finding the value $\cos 150^\circ$.

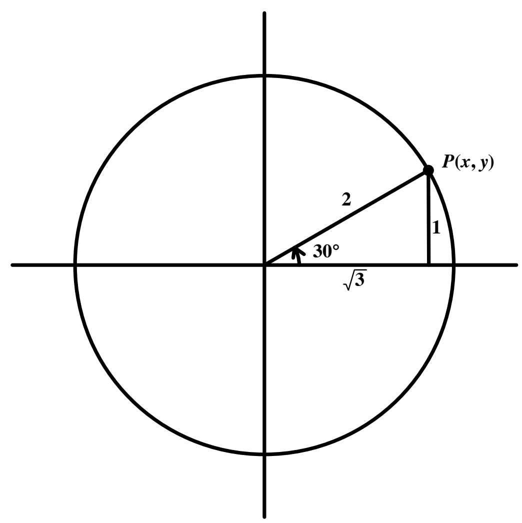
If we think of cosine as adjacent over hypotenuse, it will be impossible for us to find $\cos 150^\circ$ because there is no right triangle that has an angle measuring 150°. Indeed, our right triangle approach to the trigonometric functions is only meaningful for acute angles. But in mathematics, we encounter angles of all sizes.

3. In this problem, we will introduce a new way of thinking about trigonometric functions for angles of any size.

a. Use the diagram below to find $\cos 30^\circ$, $\sin 30^\circ$, and $\tan 30^\circ$.



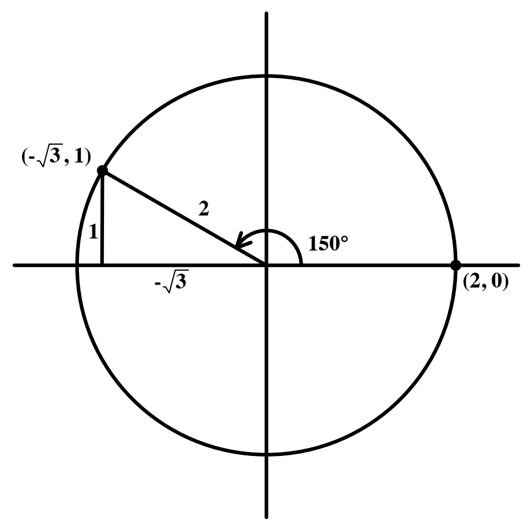
b. Notice that we could place this right triangle on a coordinate grid in the following way. We can think of the angle starting on the x-axis and rotating 30° counter clockwise. What are the coordinates of point P in this diagram?



c. You should have found that $P = (\sqrt{3}, 1)$. We might say that $x = \sqrt{3}$, $y = 1$, and $r = 2$. Can you express the values of $\cos 30^\circ$, $\sin 30^\circ$, and $\tan 30^\circ$ in terms of x, y, and r?

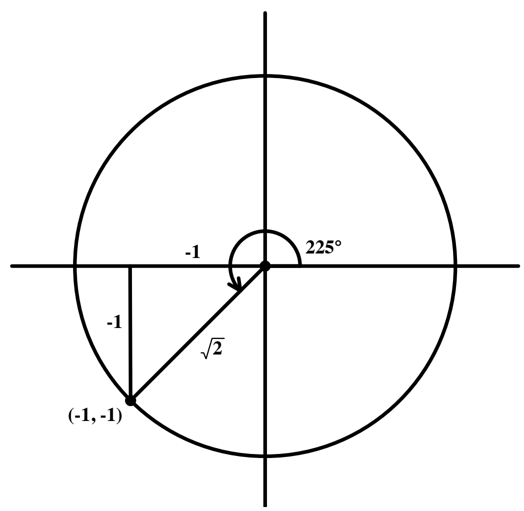
d. You should have found that $\cos 30^\circ = \frac{x}{r}$, $\sin 30^\circ = \frac{y}{r}$, and $\tan 30^\circ = \frac{y}{x}$. **Now we make the big leap:** Can you use an approach like this to find the values of $\cos 150^\circ$, $\sin 150^\circ$, and $\tan 150^\circ$? ***Note:*** If you need a hint, look ahead to part **e**.

e. Consider the diagram below. We have drawn a 150° angle starting on the positive x-axis and rotating counter clockwise to the point $(-\sqrt{3},1)$. Use this diagram to find the values of $\cos 150^\circ$, $\sin 150^\circ$, and $\tan 150^\circ$. ***Hint:*** Remember from part **c** that $\cos 30^\circ = \frac{x}{r}$, $\sin 30^\circ = \frac{y}{r}$, and $\tan 30^\circ = \frac{y}{x}$. What are *x*, *y*, and *r* in this diagram?



f. You should have found that $\cos 150^\circ = -\frac{\sqrt{3}}{2}$, $\sin 150^\circ = \frac{1}{2}$, and $\tan 150^\circ = -\frac{1}{\sqrt{3}}$. Can you use a similar approach to find the values of $\cos 225^\circ$, $\sin 225^\circ$, and $\tan 225^\circ$? ***Note:*** A hint is provided for you in part **g**. But try to figure it on your own first!

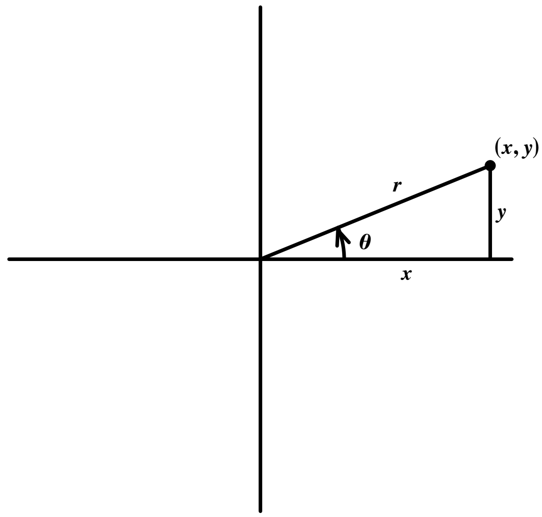
g. Here's a diagram that's helpful for answering the question from part **f**. Using this diagram, find the values of $\cos 225^\circ$, $\sin 225^\circ$, and $\tan 225^\circ$.



Answer $\cos 225^\circ = -\frac{1}{\sqrt{2}}$, $\sin 225^\circ = -\frac{1}{\sqrt{2}}$, and $\tan 225^\circ = \frac{-1}{-1} = 1$

Definitions: Circular Trigonometry

In order to evaluate trigonometric functions for angles of any size, we will now introduce a new set of definitions based on the idea we developed in the last problem. We begin all angles on the positive x-axis and rotate counterclockwise by θ. This establishes values for *x*, *y*, and *r*. We use these values to define our trig functions in the following way:



$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$

$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$

Here are some important notes about our new definitions:

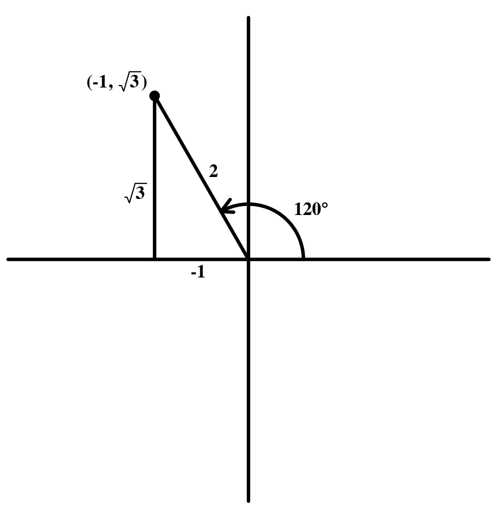
- We can choose any positive value for *r*. (Try to use whichever value is most convenient!)
- *x* and *y* could be either positive or negative, depending on which quadrant the point is in.
- We always start our angles on the positive x-axis.
- We rotate counterclockwise for positive angles and clockwise for negative angles. (Here's a way to think about this: Positive angles start off by going up and negative angles start off going down.)

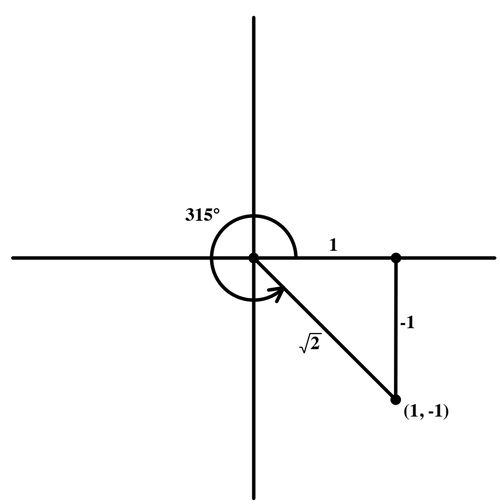
4. Use our new definitions to find each of the following without using a calculator.
Important: Draw a diagram for each example.

a. Find $\cos 120^{\circ}$

b. Find $\tan 315^{\circ}$

Answers a. $\cos 120^{\circ} = -\frac{1}{2}$ b. $\tan 315^{\circ} = \frac{-1}{1} = -1$ (See diagrams below)



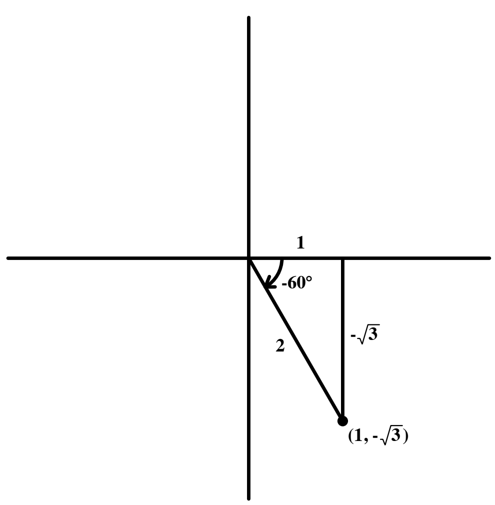


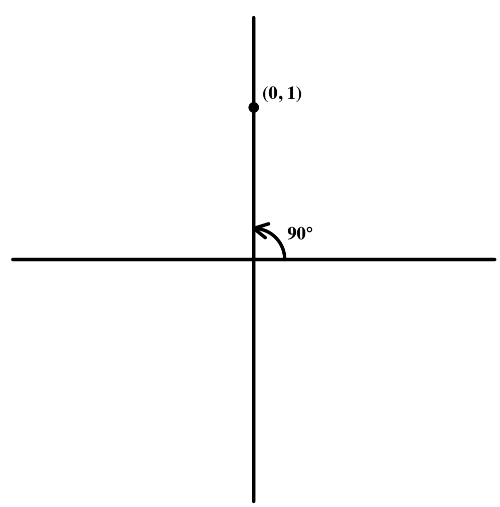
5. Use our new definitions to find each of the following without using a calculator. ***Important:*** Draw a diagram for each example.

a. Find $\sin(-60^\circ)$ ***Hint:*** When drawing the diagram for a positive angle, we rotate counterclockwise. Which way should we rotate for a negative angle?

b. Find $\sin 90^\circ$.

Answers a. $\cos 120^\circ = -\frac{1}{2}$ b. $\tan 315^\circ = \frac{-1}{1} = -1$ (See diagrams below)





6. Find each of the following. Do not use a calculator. ***Important:*** Draw a diagram for each example.

a. $\tan 30^\circ$

b. $\sin 150^\circ$

c. $\cos 135^\circ$

d. $\sin 300^\circ$

e. $\tan 225^\circ$

f. $\sin 390^\circ$

g. $\cos(-120^\circ)$

h. $\sin(840^\circ)$

i. $\cos(90^\circ)$

j. $\sin(270^\circ)$

Answers a. $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ b. $\frac{1}{2}$ c. $-\frac{1}{\sqrt{2}}$ d. $-\frac{\sqrt{3}}{2}$ e. 1 f. $\frac{1}{2}$ g. $-\frac{1}{2}$ h. $\frac{\sqrt{3}}{2}$ i. 0 j. -1

7. Find $\tan 90^\circ$. Is this value defined? ***Hint:*** Are we allowed to divide by 0?

In problem **8**, assume that $0^\circ \le \theta \le 360^\circ$

8. Solve each of the following equations. Most, but not all, have two solutions.

a. $\sin\theta = \frac{1}{2}$

b. $\cos\theta = -\frac{1}{\sqrt{2}}$

c. $\cos\theta = -\frac{\sqrt{3}}{2}$

d. $\tan\theta = -\frac{1}{\sqrt{3}}$ ***Hint:*** $-\frac{1}{\sqrt{3}} = \frac{-1}{\sqrt{3}}$ and $-\frac{1}{\sqrt{3}} = \frac{1}{-\sqrt{3}}$

e. $\sin\theta = 0$

f. $\cos\theta = 1$ ***Hint:*** How many solutions does this have?

g. $\cos\theta = 2$ ***Hint:*** What is the biggest value that $\cos\theta$ can have?

h. $\tan\theta = -\sqrt{3}$ ***Hint:*** Rewrite $-\sqrt{3}$ as a fraction.

Answers a. 30 or 150 b. 135 or 225 c. 150 or 210 d. 30 or 210 e. 0 or 180 f. 0 only g. no solution h. 120 or 300

9. Suppose $\sin\theta = \frac{3}{5}$. What are the possible values of $\cos\theta$? ***Note:*** There two solutions. Make sure you find both of them.

10. Suppose $\cos\theta = -\frac{12}{13}$. If θ is located in the third quadrant, what are the values of $\sin\theta$ and $\tan\theta$?

11. Suppose $\cos\theta = -1$. What are the values of $\sin\theta$ and $\tan\theta$?