In this problem set, we will explore how to simplify fractions with variables. As you do this assignment, think back to the knowledge of fractions that you already have.

1. Simplify each of the following fractions

   a. $\frac{4}{14}$ **b.** $\frac{24}{21}$ **c.** $\frac{6}{18}$

2. You should have found that $\frac{4}{14} = \frac{2}{7}$ because a common factor cancelled out. Notice that what we have really done is to factor the top and bottom of the fraction and then cancel the common factors: $\frac{4}{14} = \frac{2 \cdot 2}{2 \cdot 7} = \frac{2}{7}$. We can do the same thing when fractions involve variables.

   a. Simplify the fraction $\frac{15x}{20x^2}$.

   b. You should have found that $\frac{15x}{20x^2} = \frac{5x \cdot 3}{5x \cdot 4x} = \frac{3}{4x}$. Now simplify $\frac{x^2 + 7x + 12}{x^2 + 8x + 15}$.

   c. You should have found that $\frac{x^2 + 7x + 12}{x^2 + 8x + 15} = \frac{(x+3)(x+4)}{(x+3)(x+5)} = \frac{x+4}{x+5}$. Can $\frac{x+4}{x+5}$ be simplified to $\frac{4}{5}$ by cancelling the x on the top with the x on the bottom? Explain.

   d. We **cannot** simplify $\frac{x+4}{x+5}$ to $\frac{4}{5}$ because the x is **not a factor** of either the top or the bottom. Now simplify $\frac{x^2 - 6x - 7}{x^2 + 10x + 9}$.

3. A fraction in which both the numerator and the denominator are polynomials is called a *rational expression*. Simplify each of the following rational expression. ***Note:*** Answers are provided at the end of this problem.

a. $\dfrac{6x^2}{7x^3}$

b. $\dfrac{20x^4}{10x^2}$

c. $\dfrac{x^2+5x}{x^2+10x}$

d. $\dfrac{2x^2-14x}{3x-21}$

e. $\dfrac{x^2+4x-32}{x^2-8x+16}$

f. $\dfrac{x^2-9}{x^2+10x+21}$

g. $\dfrac{2x^2 + x - 3}{2x^2 + 7x + 6}$

h. $\dfrac{2x^2 + 12x + 16}{2x^2 - x - 10}$

i. $\dfrac{x^4 + 2x^2 - 35}{x^3 + 2x^2 - 5x - 10}$

j. $\dfrac{2x^4 - 6x^3 + 8x^2 - 24x}{2x^3 - 4x^2 - 6x}$

Answers a. $\dfrac{6}{7x}$ **b.** $2x^2$ **c.** $\dfrac{x+5}{x+10}$ **d.** $\dfrac{2x}{3}$ **e.** $\dfrac{x+8}{x-4}$ **f.** $\dfrac{x-3}{x+7}$ **g.** $\dfrac{x-1}{x+2}$ **h.** $\dfrac{2x+8}{2x-5}$

i. $\dfrac{x^2+7}{x+2}$ **j.** $\dfrac{x^2+4}{x+1}$

4. In this problem, we will investigate how to multiply and divide rational expressions.

    a. Multiply $\frac{3}{4} \cdot \frac{7}{2}$

    b. You should have found that $\frac{3}{4} \cdot \frac{7}{2} = \frac{21}{8}$. Now multiply $\dfrac{3x}{5} \cdot \dfrac{2x}{x+6}$.

    c. You should have found that $\dfrac{3x}{5} \cdot \dfrac{2x}{x+6} = \dfrac{6x^2}{5x+30}$. Now multiply $\frac{28}{45} \cdot \frac{18}{21}$.

    d. You should have found that $\frac{28}{45} \cdot \frac{18}{21} = \frac{4 \cdot 7}{5 \cdot 9} \cdot \frac{2 \cdot 9}{3 \cdot 7} = \frac{4}{5} \cdot \frac{2}{3} = \frac{8}{15}$. Notice that it is sometimes easier to factor and cancel before you multiply. Now multiply $\dfrac{x^2+10x+16}{x^2-10x+9} \cdot \dfrac{3x-27}{x^2+8x}$.

5. In the last problem, you should have found that $\dfrac{x^2 + 10x + 16}{x^2 - 10x + 9} \cdot \dfrac{3x - 27}{x^2 + 8x} = \dfrac{3x + 6}{x^2 - x}$. Now do each of the following multiplications. ***Hint:*** Some problems are easier if you factor first.

a. $\dfrac{3}{7x^2} \cdot \dfrac{5}{6}$

b. $\dfrac{4x}{15} \cdot \dfrac{5}{2x}$

c. $\dfrac{x - 4}{x + 7} \cdot \dfrac{2x}{5}$

d. $\dfrac{x + 8}{x - 6} \cdot \dfrac{x - 8}{x + 10}$

e. $\dfrac{x^2 - 9}{x^2 - 4} \cdot \dfrac{x^2 + 9x + 14}{x^2 - 7x + 12}$

f. $\dfrac{x^4 + x^2 - 56}{x - 9} \cdot \dfrac{x + 3}{x^2 + 8}$

Answers a. $\dfrac{5}{14x^2}$ b. $\dfrac{2}{3}$ c. $\dfrac{2x^2 - 8x}{5x + 35}$ d. $\dfrac{x^2 - 64}{x^2 + 4x - 60}$ e. $\dfrac{x^2 + 10x + 21}{x^2 - 6x + 8}$

f. $\dfrac{x^3 + 3x^2 - 7x - 21}{x - 9}$

6. More problems! Simplify.

a. $\dfrac{2x+4}{6}$

b. $\dfrac{x^2-1}{2x-2}$

c. $\dfrac{3x-3}{x^2+4x-5}$

d. $\dfrac{x^2+5x}{x^2}$

e. $\dfrac{x^2 + 5x}{x^2 + 5}$

f. $\dfrac{x^4 - x^2}{x^3 - 5x^2 - 6x}$

g. $\dfrac{2x^2 - 8}{x^3 - 2x^2 - x + 2}$

h. $\dfrac{2x^2 y}{3y^2} \cdot \dfrac{9x^3 y^2}{18x}$

i. $\dfrac{6a^2b^2}{9b^4} \cdot 6ab$

j. $\dfrac{x}{2} \cdot \dfrac{x^2-4}{x^2-x-6} \cdot \dfrac{x-3}{x+7}$

l. $\dfrac{4x^2-8x-21}{4x^2+11x-3} \cdot \dfrac{4x^2-9x+2}{2x^2-9x+7}$

Answers a. $\dfrac{x+2}{3}$ **b.** $\dfrac{x+1}{2}$ **c.** $\dfrac{3}{x+5}$ **d.** $\dfrac{x+5}{x}$ **e.** $\dfrac{x^2+5x}{x^2+5}$ (already simplified)

f. $\dfrac{x^2-x}{x-6}$ **g.** $\dfrac{2x+4}{x^2-1}$ **h.** $\dfrac{x^4y}{3}$ **i.** $\dfrac{4a^3}{b}$ **j.** $\dfrac{x^2-2x}{2x+14}$ **l.** $\dfrac{2x^2-x-6}{x^2+2x-3}$

7. Determine whether each of the following statements is true or false

a. $\dfrac{x^3 + 4}{x^3 + 7} = \dfrac{4}{7}$

b. $\dfrac{3x + 2}{6x + 1} = \dfrac{x + 2}{2x + 1}$

c. $\dfrac{x^{12}}{x^8} = \dfrac{x^3}{x^2}$

d. $\dfrac{2}{3}\left(\dfrac{x - 4}{x + 1}\right) = \dfrac{2(x - 4)}{3(x + 1)}$

Answers a. False   b. False   c. False   d. True