

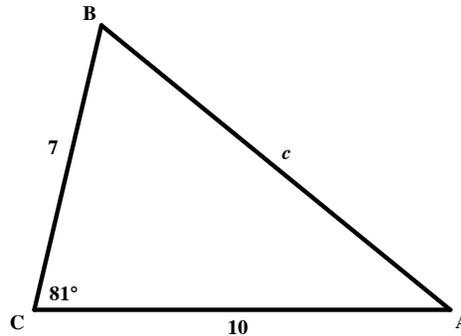
Algebra 2/Pre-Calculus

Name _____

Introduction: Law of Cosines (Day 3, Geometric Trigonometry)

In this problem set, we will introduce The Law of Cosines. (Don't worry if you don't know that that is yet – it will be explained as you work through these problems.)

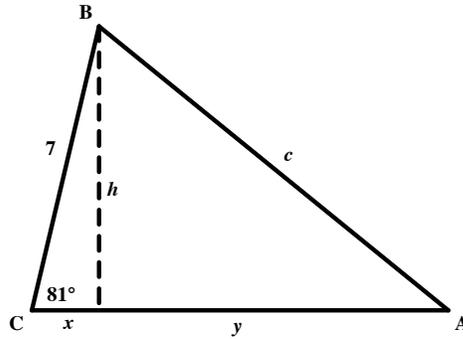
1. Consider the triangle below. Our goal is to find the length of side c .



- a. This triangle is almost (but not quite) a right triangle. Use the Pythagorean theorem to estimate the length of side c .

- b. If we use the Pythagorean theorem, we get $7^2 + 10^2 \approx c^2$, so $c \approx 12.21$. Is the length of c bigger or smaller than 12.21? How do you know?

- c. Since $\angle C$ is acute, we can conclude that c is smaller than 12.21. The rest of this problem will be devoted to finding the exact value for side c . We begin by dropping an altitude to side b , as shown below. *Note:* Even though it isn't labeled on the diagram, remember that side b has a length of 10. This means $x + y = 10$.

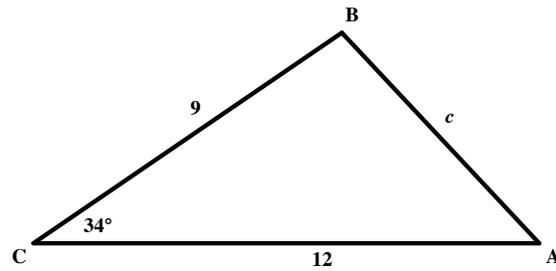


Can you find the length of h ? What about the length of x ? The length of y ? The length of c ?

Solution

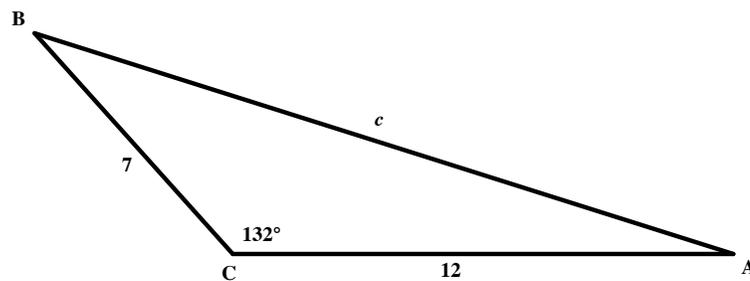
Since $\sin 81^\circ = \frac{h}{7}$ and $\cos 81^\circ = \frac{x}{7}$, we know that $h = 7 \sin 81^\circ = 6.91$ and $x = 7 \cos 81^\circ = 1.10$. This means that $y = 10 - 1.10 = 8.90$. Using Pythagorean theorem again, we get $h^2 + y^2 = c^2$, so $6.91^2 + 8.90^2 = c^2$. Solving for c , we get 11.27.

2. Use the method from problem 1 to find the length of side c in the triangle below.



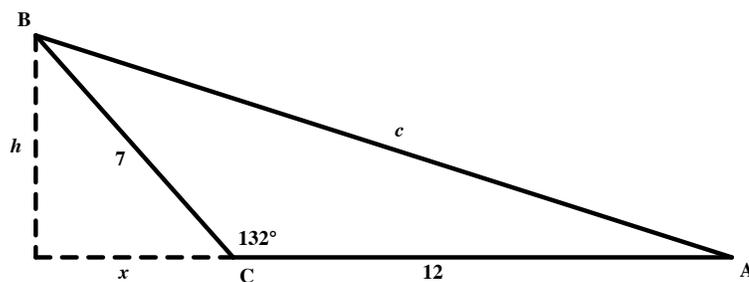
Answer 6.78

3. Consider the triangle below. Again, our goal is to find the length of side c .



- a. If we "pretended" that $\angle C$ was a right triangle, we would get the equation $7^2 + 12^2 = c^2$. What value would this give us for c ? Is the actual value of c bigger or smaller than this estimate?

- b. Our goal is to find the exact value of side c . To do this, we will drop an altitude, as shown below.

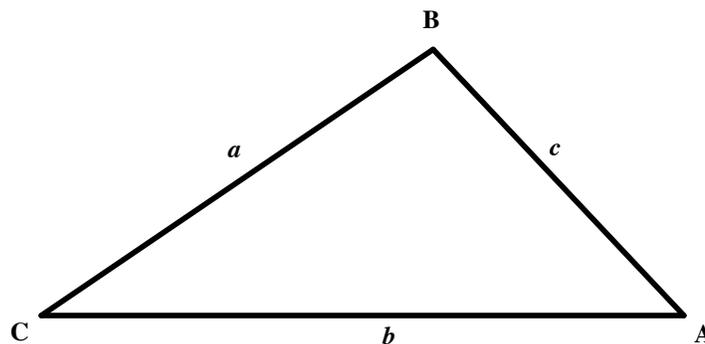


Find the values of x and h . Then find the value of c .

Answers a. c is bigger than our estimate because $\angle C$ is obtuse

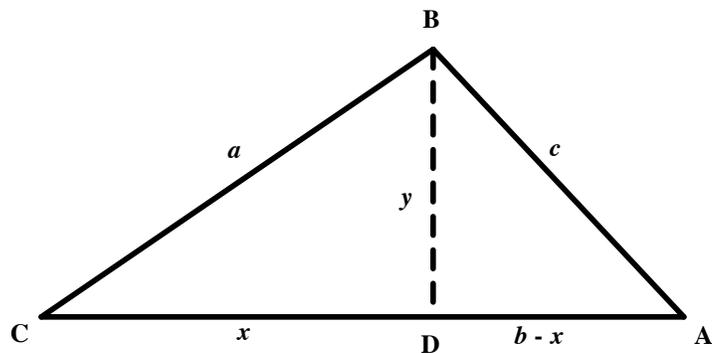
b. $x = 4.68$, $h = 5.20$, $c = 17.48$

4. In this problem, we will generalize the approach we used in the last few problems. Consider the triangle pictured below.

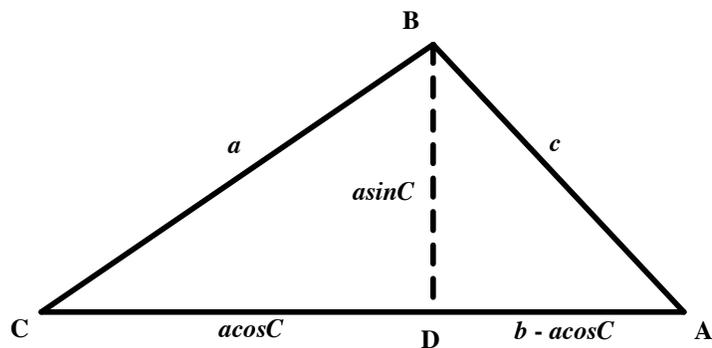


- a. Our goal is to find a formula for the length of side c in terms of angle C and sides a and b . Can you find it? **Note:** There are hints in the later parts of this problem. But try in on your own first.

- b. As we've seen in the last few problems, it is very helpful to begin by dropping an altitude. See if you can use this suggestion to find the length of side c . Your answer will be in terms of angle C and sides a and b . **Hint:** Use trig functions to find x and y in terms of side a and angle C .



- c. You should have found that $x = a \cos C$ and $y = a \sin C$. (Make sure you can see why this is the case!) Now find the value of side c . **Hint:** Use the Pythagorean theorem on $\triangle ABD$. **Reminder:** $\cos^2 \theta + \sin^2 \theta = 1$.

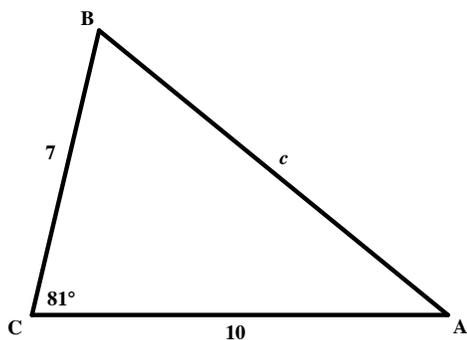


You should have found the following equation for side c :

$$c^2 = a^2 + b^2 - 2ab\cos C$$

We call this rule the **Law of Cosines**. It is very helpful when solving problems involving triangles such as the ones from the beginning of this handout.

5. Use the law of cosines to solve for c in the triangle below. **Note:** This is the same as the first problem from this handout, but we can now solve it much more easily!



Solution We solve $c^2 = 7^2 + 10^2 - 2(7)(10)\cos 81$ to find $c = 11.27$