

Algebra 2/Pre-Calculus

Inverse Trig Equations (Day 1, Inverse and Geometric Trig)

Name _____

In this handout, we will begin solving trig equations by using inverse trig functions. We will need to use our calculators and our answers will be approximate.

1. Our goal is to solve the equation $\sin \theta = \frac{1}{5}$ on $0^\circ \leq \theta \leq 360^\circ$.
 - a. Draw two possible triangles representing the two possible values for θ . **Hint:** Which quadrants are the triangles in?
 - b. The triangles you drew above are not special triangles, so we will need to use our calculator to find the angles. To do this, enter $\sin^{-1}(1/5)$ on your calculator. **Note:** You will need to use the 2nd button. **Important:** Press the MODE button on your calculator and make sure you are in DEGREE mode.
 - c. You should have found that $\theta = 11.5^\circ$. (If you didn't get this, double check that the calculator is in DEGREE mode.) This is the solution in the first quadrant. Use your diagram to find the angle in the second quadrant. **Hint:** What is the relationship between these two angles?
 - d. You should have found that the other angle was $\theta = 168.5^\circ$. (The two angles were supplementary.) Now solve the equation $\sin \theta = \frac{2}{7}$ on $0^\circ \leq \theta \leq 360^\circ$. (Use a procedure similar to what you did in parts a-c. You should get two solutions.)

2. Here's an equation involving cosine: $\cos\theta = \frac{3}{4}$ on $0^\circ \leq \theta \leq 360^\circ$.
- Draw two possible triangles representing the two possible values for θ . **Hint:** Which quadrants are the triangles in?
 - The triangles you drew above are not special triangles, so we will need to use our calculator to find the angles. To do this, enter $\cos^{-1}(3/4)$ on your calculator. **Note:** You will need to use the 2nd button. **Important:** Press the MODE button on your calculator and make sure you are in DEGREE mode.
 - You should have found that $\theta = 41.4^\circ$. (If you didn't get this, double check that the calculator is in DEGREE mode.) This is the solution in the first quadrant. Use your diagram to find the angle in the fourth quadrant.
 - The easiest way to describe the other angle is $\theta = -41.4^\circ$. (The two angles were supplementary.) But we were asked for solutions on $0^\circ \leq \theta \leq 360^\circ$. How do you turn -41.4° in to a solution on $0^\circ \leq \theta \leq 360^\circ$?
 - You should have found that the two solutions were $\theta = 41.4^\circ$ and $\theta = 318.6^\circ$. Now use a similar procedure to solve the equation $\cos\theta = \frac{6}{7}$ on $0^\circ \leq \theta \leq 360^\circ$.

3. Solve $\sin \theta = 0.7$ on $0^\circ \leq \theta \leq 360^\circ$. **Hint:** When drawing your triangles, it may be helpful to rewrite 0.7 as $\frac{0.7}{1}$.

4. Solve $\cos \theta = 0.23$ on $0^\circ \leq \theta \leq 360^\circ$.

5. Here's a harder equation involving cosine: $\cos \theta = -\frac{8}{9}$ on $0^\circ \leq \theta \leq 360^\circ$.

a. Draw two possible triangles representing the two possible values for θ . **Hint:** Which quadrants are the triangles in? Be careful! This time, the ratio is negative.

b. You should have found that the two triangles were in quadrants two and three. (These are the quadrants in which x is negative.) Use $\cos^{-1}(-8/9)$ to find one of the angles. Which quadrant is this angle in?

c. You should have found that the second quadrant angle was $\theta = 152.7^\circ$. Now find the angle in the third quadrant. **Hint:** Think about the relationship between the two angles.

d. The other angle is most easily described as $\theta = -152.7^\circ$. To put this angle on $0^\circ \leq \theta \leq 360^\circ$, we rewrite it as $\theta = 207.3^\circ$. (Because $-152.7^\circ + 360^\circ = 207.3^\circ$.)
Now solve the equation $\cos \theta = -\frac{1}{4}$ on $0^\circ \leq \theta \leq 360^\circ$.

6. Here's another sine equation: $8\sin\theta + 3 = 2$ on $0^\circ \leq \theta \leq 360^\circ$.

a. Start by simplifying the equation. Can you isolate $\sin\theta$?

b. You should have found that $\sin\theta = -\frac{1}{8}$. Draw two possible triangles representing the two possible values for θ . **Hint:** Which quadrants are the triangles in? Be careful! The ratio is negative.

c. You should have found that the two triangles were in quadrants three and four. (These are the quadrants in which y is negative.) Use $\sin^{-1}(-1/8)$ to find one of the angles. Which quadrant is this angle in?

d. You should have found that the fourth quadrant angle was $\theta = -7.2^\circ$. Note that we can also rewrite this as $\theta = 352.8^\circ$. Now find the angle in the third quadrant. **Hint:** Think about the relationship between the two angles.

e. You should have found that the other angle was $\theta = 187.2^\circ$. (Because $7.2^\circ + 180^\circ = 187.2^\circ$. Make sure you can see why this is true. This one is the trickiest case!)

Now solve the equation $\sin\theta = -.78$ on $0^\circ \leq \theta \leq 360^\circ$.

7. In the last group of problems, we were solving for $0^\circ \leq \theta \leq 360^\circ$. In this problem, we will solve equations in which θ can have any value. **Hint:** When writing your answers, you in need to include “ $+360^\circ N$.” **Note:** Answers are provided at the end of this problem.

a. $\sin \theta = \frac{5}{6}$

b. $7 \cos \theta + 3 = 5$

c. $\sin \theta = -0.88$

d. $7 \cos \theta + 3 = 5$

Answers

a. $\theta = 56.44^\circ + 360^\circ N$ or $\theta = 123.56^\circ + 360^\circ N$ b. $\theta = 73.40^\circ + 360^\circ N$ or $\theta = 286.60^\circ + 360^\circ N$

c. $\theta = 298.36^\circ + 360^\circ N$ or $\theta = 241.64^\circ + 360^\circ N$ d. $\theta = 107.46^\circ + 360^\circ N$ or $\theta = 252.54^\circ + 360^\circ N$

Sometimes (like in the example below) trigonometric equations will “look like” quadratics. We call these types of equations quadratics forms and we will solve them using the same techniques that we use on quadratic equations (especially factoring).

Example: Find all solutions to the equation $5\sin^2 x - 3\sin x = 0$ for $0^\circ \leq x \leq 360^\circ$.

Solution:

$$5\sin^2 x - 3\sin x = 0$$

$$\sin x(5\sin x - 3) = 0$$

$$\sin x = 0 \text{ or } 5\sin x - 3 = 0$$

$$\sin x = 0 \text{ or } \sin x = \frac{3}{5}$$

$$x = 0^\circ \text{ or } x = 180^\circ \text{ or } x = 36.87^\circ \text{ or } x = 143.13^\circ$$

Note: We need to use the calculator in the step where we find the solutions to $\sin x = \frac{3}{5}$.

8. Solve each of the following equations for $0^\circ \leq x \leq 360^\circ$. Use your calculator only when necessary.

a. $7\cos^2 x - 2\cos x = 0$

b. $9\sin^2 x - 4 = 0$ **Hint:** How do you solve the equation $9u^2 - 4 = 0$?

c. $6\sin^2 x + \sin x - 2 = 0$ *Hint:* How do you factor $6u^2 + u - 2$?

d. $5 - 5\cos^2 x = 3\sin x$ *Hint:* Use the fact that $\cos^2 x = 1 - \sin^2 x$.

e. $4\sin^2 x = 3 + 3\cos x$