Algebra 2/Pre-Calculus

Name_____

More Applications of Normal Distributions (Day 11, Statistics)

For all probability problems on this handout, use the normal curve to approximate the probabilities.

1. A bent coin lands heads 65% of the time and tails 35% of the time. We flip the coin 30 times.

a. Find the mean and standard deviation for the number of heads.

b. Which numbers (of heads) are within one standard deviation of the mean?

c. What is the probability that the number of heads is within one standard deviation of the mean? ***Hint:*** What should you enter into wolframalpha to answer this question?

d. Using the normal curve, estimate the probability that the number of heads is within one standard deviation of the mean.

e. How did your answer for parts **c** and **d** compare? Were they close? Were they exactly the same? Explain.

Answers a. mean = 19.5, standard deviation = 2.612 b. 17, 18, 19, 20, 21, 22 c. 0.7499 (Using wolframalpha, find $(0.65h + 0.35t)^{30}$. Then add up the relevant coefficients.)

d. 0.6827 e. They aren't exactly the same. The probability histogram is for this situation is similar to the normal curve, but it isn't exactly the normal curve. Hence, the answer in part **c** is more accurate, whereas the answer in part **d** is an approximation.

2. Another bent coin lands heads 53% of the time. We flip this coin 50 times.

a. Find the mean and standard deviation for the number of heads.

b. Which numbers (of heads) are within 0.5 standard deviations of the mean?

c. What is the probability that the number of heads is within 0.5 standard deviation of the mean? ***Hint:*** Use wolframalpha.

d. Using the normal curve, estimate the probability that the number of heads is within 0.5 standard deviations of the mean.

e. How did your answer for parts **c** and **d** compare? Which one is more accurate? Explain.

Answers a. mean = 26.5, standard deviation = 3.529 b. 25, 26, 27, 28 c. 0.4283 d. 0.3829 e. The value found in part c is more accurate. The normal curve is an approximation.

3. Which is more accurate: Using the normal curve to approximate the number of heads when a coin is flipped 30 times or using the normal curve to approximate the number of heads when a coin is flipped 200 times? Explain.

Answer In general, as the coin is flipped more times, the accuracy of estimating probabilities using the normal curve increases

4. Suppose a fair coin is flipped 200 times.

a. What is the probability of getting exactly 100 heads? ***Note:*** Do not use wolframalpha.

b. What is the probability that the number of heads will be either 99, 100, or 101? ***Note:*** Again, no wolframalpha.

c. Now suppose we want to know the probability that the number of heads is between 90 and 110. This calculation is quite tedious, even with the help of wolframalpha. So instead of doing it directly, we will use the normal curve to approximate the answer. To do this, we begin by determining how many standard deviations below the mean is 90 and how many standard deviations above the mean is 110.

d. Use your calculator to find the probability that the number of heads is between 90 and 110. ***Hint:*** What should you use as your upper and lower bounds?

e. Use a similar procedure to determine the probability that the number of heads is between 95 and 120.

f. Determine the probability that the number of heads is at least 115. ***Hint:*** Just use 5 as the upper bound.

g. Determine the probability that the number of heads is at most 93.

Answers a. 5.63% b. 16.79% c. mean = 100, standard deviation = 7.071, 90 is 1.414 standard deviations below the mean and 110 is 1.414 standard deviations above the mean.

d. 84.26% e. 75.79% f. 1.69% g. 16.11%

5. An unfair coin lands heads 30% of the time. We flip this coin 300 times.

a. What is the probability of getting exactly 95 heads? ***Note:*** Do not use wolframalpha.

b. What is the probability that the number of heads will be either 93, 94, or 95? ***Note:*** Again, no wolframalpha.

c. What is the probability that the number is heads is between 85 and 100? ***Hint:*** Use the technique from the last problem to estimate the answer by using the normal curve.

d. What is the probability that the number of heads is between 90 and 110? ***Note:*** Estimate using the normal curve.

e. What is the probability that the number of heads is at least 105? ***Note:*** Estimate using the normal curve.

Answers a. 4.06% b. 13.07% c. 63.18% d. 49.39% e. 3.00%