Algebra 2/Pre-Calculus

Name_____

More Equation Solving (Circular Trig, Day 6)

In this handout we will continue using the circular trig definitions (x, y, and r, rather than adjacent, opposite, and hypotenuse). All of the problems on this handout should be done **without the aid of a calculator**.

1. Find each of the following.

a. $\sin 390^\circ$

b. $\cos 405^\circ$

c. $\sin 510^\circ$

d. $\cos 570^\circ$

e. $\sin 540^\circ$

f. $\sin 810^\circ$

g. $\cos 945^\circ$

h. $\sin 1380^\circ$

i. $\tan(-390)^\circ$

j. $\tan 450^\circ$

Answers (Not yet double checked!)

a. $\frac{1}{2}$ b. $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ c. $\frac{1}{2}$ d. $-\frac{\sqrt{3}}{2}$ e. 0 f. 1 g. $-\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ h. $-\frac{\sqrt{3}}{2}$ i. $-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

j. undefined

2. Look back at problem **1**. Describe any observations that you made.

3. Consider the equation $\sin\theta = \frac{1}{2}$.

a. Find all solutions to the equation $\sin\theta = \frac{1}{2}$ when $0^\circ \le \theta < 360^\circ$.

b. You should have found that $\theta = 30^\circ$ or $\theta = 150^\circ$. Are there any other solutions?

Hint: What if $\theta > 360^\circ$

c. Is $\theta = 390^\circ$ a solution to the equation $\sin\theta = \frac{1}{2}$? What about $\theta = 510^\circ$?

d. $\theta = 390^\circ$ and $\theta = 510^\circ$ are both solutions to the equation $\sin\theta = \frac{1}{2}$. List four more solutions.

e. Here are four more solutions to the equation $\sin\theta = \frac{1}{2}$: 750°, 870°, 1110°, and 1230°. Are there any negative values of θ that are solutions to this equation? List some.

4. Here's a way of describing all solutions the equation $\sin\theta = \frac{1}{2}$:

$\theta = 30^\circ + 360^\circ N$ or $\theta = 150^\circ + 360^\circ N$ where N is an integer.

a. Explain why this works.

b. What values of θ do you get when $N = 1$?

c. When $N = 1$, $\theta = 390^\circ$ or $\theta = 510^\circ$. (Recall that both of these are solutions to the equation $\sin\theta = \frac{1}{2}$.) What values of θ do you get when $N = -1$?

5. Consider the equation $\cos\theta = -\frac{1}{2}$.

a. Find all solutions for the equation $\cos\theta = -\frac{1}{2}$ such that $0^\circ \le \theta < 360^\circ$.

b. Find all solution for the equation $\cos\theta = -\frac{1}{2}$. (All solutions, not just solutions on $0^\circ \le \theta < 360^\circ$.)

c. List six solutions to the equation $\cos\theta = -\frac{1}{2}$.

Answer a. $\theta = 120^\circ$ or $\theta = 240^\circ$ b. $\theta = 120^\circ + 360^\circ N$ or $\theta = 240^\circ + 360^\circ N$

c. There are many solutions. Here are a few of them: 120°, 240°, 480°, 600°, 840°, 960°, -120°, -240°, etc.

6. Consider the equation $\sin\theta = 1$.

a. Find all solutions for the equation $\sin\theta = 1$ such that $0^\circ \le \theta < 360^\circ$.

b. Find all solution for the equation $\sin\theta = 1$. (All solutions, not just solutions on $0^\circ \le \theta < 360^\circ$.)

c. List six solutions to the equation $\sin\theta = 1$.

d. The equation $\sin\theta = 1$ only had one solution on $0^\circ \le \theta < 360^\circ$. List some other trig equations that only have one solution on $0^\circ \le \theta < 360^\circ$.

Answer a. $\theta = 90^\circ$ b. $\theta = 90^\circ + 360^\circ N$ c. There are many solutions. Here are a few of them: $\theta = 90^\circ$, 450°, 810°, 1170°, -270°, -630°, etc. d. $\sin\theta = -1$, $\cos\theta = 1$, $\cos\theta = -1$

7. Find all solutions to each of the following equations. ***Note:*** Answers are provided at the end of this problem.

a. $\sin\theta = \frac{\sqrt{3}}{2}$

b. $\cos\theta = \frac{\sqrt{2}}{2}$

c. $\sin\theta = -\frac{1}{2}$

d. $\cos\theta = 0$

e. $\cos\theta = -1$

f. $\tan\theta = \sqrt{3}$

g. $\tan\theta = -1$

h. $\sin\theta = -\frac{\sqrt{3}}{2}$

Answers

a. $\theta = 60^\circ + 360^\circ N$ or $\theta = 120^\circ + 360^\circ N$ b. $\theta = 45^\circ + 360^\circ N$ or $\theta = 315^\circ + 360^\circ N$

c. $\theta = 210^\circ + 360^\circ N$ or $\theta = 330^\circ + 360^\circ N$ d. $\theta = 90^\circ + 360^\circ N$ or $\theta = 270^\circ + 360^\circ N$

e. $\theta = 180^\circ + 360^\circ N$ f. $\theta = 60^\circ + 360^\circ N$ or $\theta = 240^\circ + 360^\circ N$

g. $\theta = 135^\circ + 360^\circ N$ or $\theta = 315^\circ + 360^\circ N$ h. $\theta = 240^\circ + 360^\circ N$ or $\theta = 300^\circ + 360^\circ N$

8. In this problem, we will combine our knowledge of equation solving with our understanding of trigonometric identities.

a. Simplify $\sin(180^\circ - \theta)$.

b. You should have found that $\sin(180^\circ - \theta) = \sin\theta$. Now solve the equation $\sin(180^\circ - \theta) = \frac{1}{2}$.

c. Since $\sin(180^\circ - \theta) = \sin\theta$, the equation $\sin(180^\circ - \theta) = \frac{1}{2}$ can be simplified to $\sin\theta = \frac{1}{2}$ which has solutions of $\theta = 30^\circ + 360^\circ N$ or $\theta = 150^\circ + 360^\circ N$.

Now simplify $\cos(180^\circ - \theta)$.

d. You should have found that $\cos(180^\circ - \theta) = -\cos\theta$. Now solve the equation $\cos(180^\circ - \theta) = \frac{\sqrt{2}}{2}$.

e. Since $\cos(180^\circ - \theta) = -\cos\theta$, the equation $\cos(180^\circ - \theta) = \frac{\sqrt{2}}{2}$ can be simplified to $\cos\theta = -\frac{\sqrt{2}}{2}$ which has solutions of $\theta = 135^\circ + 360^\circ N$ or $\theta = 225^\circ + 360^\circ N$.

Now simplify $\cos(\theta + 90^\circ)$.

f. Now solve the equation $\cos(\theta + 90^\circ) = \frac{\sqrt{3}}{2}$.

g. Solve the equation $\sin(90^\circ - \theta) = 0$.

h. Solve the equation $\sin(\theta + 180^\circ) = 1$.

i. Solve the equation $\sin(-\theta) = \frac{1}{2}$.

Some answers (Not yet double checked!)

e. $\cos(\theta + 90^\circ) = -\sin\theta$ f. $\theta = 240^\circ + 360^\circ N$ or $\theta = 300^\circ + 360^\circ N$

g. $\theta = 90^\circ + 360^\circ N$ or $\theta = 270^\circ + 360^\circ N$ h. $\theta = 270^\circ + 360^\circ N$

i. $\theta = 210^\circ + 360^\circ N$ or $\theta = 330^\circ + 360^\circ N$