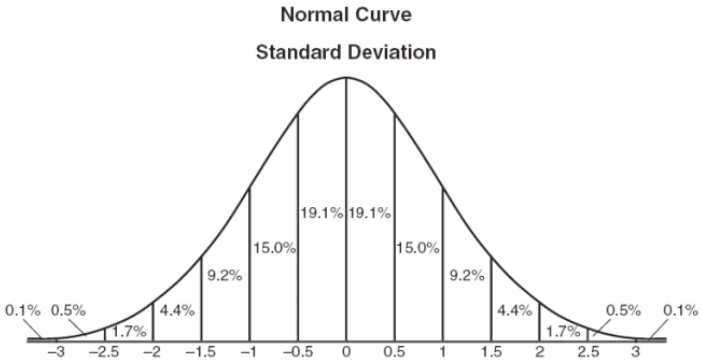
Algebra 2/Pre-Calculus

Name_____

More Practice with Normal PDF and CDF (Day 13, Statistics)

In this problem set, we will continue exploring Normal PDF and Normal CDF.

1. Use the bell curve below to answer the following each of the following questions **without using your calculator**. ***Note:*** Answers are provided at the end of this problem.



a. What percent of the values are within one standard deviation of the mean?

b. What percent of the values are within half a standard deviation of the mean?

c. What percent of the values are at least two standard deviations above the mean?

d. Find normalcdf(-0.5, 1, 0, 1). ***Remember:*** No calculators!

e. Find normalcdf(0, 2, 0, 1).

f. Suppose you flip a fair coin 100 times. Find the probability that the number of heads will be between 50 and 55. ***Note:*** Use the diagram of the normal curve given above to estimate the answer. ***Hint:*** Start by finding the mean and standard deviation for the number of heads from 100 flips.

g. Find normalcdf(50, 55, 50, 5). ***Hint:*** How is this related to the last question?

h. Find normalcdf(45, 55, 50, 5). ***Hint:*** How is this related to flipping a fair coin 100 times?

i. Find normalcdf(450, 480, 450, 30). ***Hint:*** Start by identifying the mean and the standard deviation.

j. Find normalcdf(1010, 1030, 1000, 20).

Answers a. $15\% + 19.1\% + 19.1\% + 15\% = 68.2\%$ b. $19.1\% + 19.1\% = 38.2\%$

c. $1.7\% + 0.5\% + 0.1\% = 2.3\%$ d. $19.1\% + 19.1\% + 15\% = 53.2\%$

e. $19.1\% + 15\% + 9.2\% + 4.4\% = 47.7\%$ f. $19.1\% + 15\% = 34.1\%$ g. $19.1\% + 15\% = 34.1\%$

h. $15\% + 19.1\% + 19.1\% + 15\% = 68.2\%$ i. $19.1\% + 15\% = 34.1\%$ j. $15\% + 9.2\% = 24.2\%$

Use Normal PDF or Normal CDF to answer the remaining questions in this problem set.

2. A certain brand of light switch will switch 1,000 times, on average, in its lifetime with a standard deviation of 20.

a. What is the probability that a switch you buy will switch exactly 990 times?

b. What is the probability that a switch you buy will switch exactly 998, 999, or 1000 times?

Answers a. $\text{normalpdf}(990,1000,20) = 1.76\%$

b. $\text{normalpdf}(998,1000,20) + \text{normalpdf}(999,1000,20) + \text{normalpdf}(1000,1000,20) = 5.97\%$

3. On a college exam in nursing, scores are normally distributed with a mean of 61.5 and a standard deviation of 10.7. The college requires that you score at least 75 to get a merit scholarship. What percent of students get the necessary score?

Answer $\text{normalcdf}(75,100,61.5,10.7) = 10.4\%$

4. A brand of refrigerator motor has a life expectancy that is normally distributed with a mean of 18 years and a standard deviation of 3 years.

a. What percentage of these motors last more than 25 years?

b. What percentage last between 15 and 21 years?

Answers a. $\text{normalcdf}(25,1000,18,3) = 0.98\%$ b. $\text{normalcdf}(15,21,18,3) = 68.3\%$

5. The SAT exam is scaled so that the mean score on each section for all high school students in the country should be exactly 500 and the standard deviation exactly 100. What percentage of students earn a score of exactly 500?

Answer $\text{normalpdf}(500,500,100) = 0.399\%$

6. The Appaloosa Apple Cider Company fills its containers automatically. The quantity delivered is approximately normally distributed with an average volume of 32.7 oz. and a standard deviation of 0.25 oz.

a. What percentage of containers overflow at the top, which occurs when 33.4 oz or more are delivered?

b. The company prints "32 fluid ounces" on the label. What percentage of containers fulfill this promise exactly?

c. What percentage contain between 31 and 33 oz?

Answers a. $\text{normalcdf}(33.4,1000,32.7,0.25) = 0.26\%$

b. $\text{normalpdf}(32,32.7,0.25) = 3.17\%$ c. $\text{normalcdf}(31,33,32.7,0.25) = 88.5\%$

7. In order to justify installing windmills to generate electricity there should be winds of at least 10.5 MPH 90% of the time. Suppose winds on Mt. Wachusett have an average speed of 18 MPH with a standard deviation of 5.2 MPH. Is this site suitable for a wind generator?

Answer $\text{normalcdf}(10.5,1000,18,5.2) = 92.5\%$

8. In a recent homework, you were asked "If you rolled 240 number cubes and got only 20 sixes, would that be a surprising result or within reasonable limits?" Find the approximate probability that the number of sixes recorded is 20 or fewer.

Answer $\text{normalcdf}(0,20,40,5.7735) = 0.0266\%$

9. For the following number of coin flips, find the approximate probability of getting 49% or fewer heads. For example, when flipping 200 coins, this is asking for the probability of getting 98 or fewer heads.

a. 200 coin flips

b. 400 coin flips

c. 900 coin flips

d. 2500 coin flips

e. 10,000 coin flips

f. What do you notice about these probabilities as the number of trials increases? Explain why this occurs.

Answer $\text{normalcdf}(0,98,100,7.0711) = 38.86\%$ b. $\text{normalcdf}(0,196,200,10) = 34.46\%$ c. $\text{normalcdf}(0,441,450,15) = 27.43\%$ d. $\text{normalcdf}(0,1225,1250,25) = 15.87\%$ e. $\text{normalcdf}(0,4900,5000,50) = 2.28\%$ f. As the number of flips increases, more and more of the values are "closer" to the mean. Specifically, more and more of the results are within 1% of the mean. (Notice that for 10,000 flips, nearly all of the results are within 1% of the mean.)