Algebra 2/Pre-Calculus

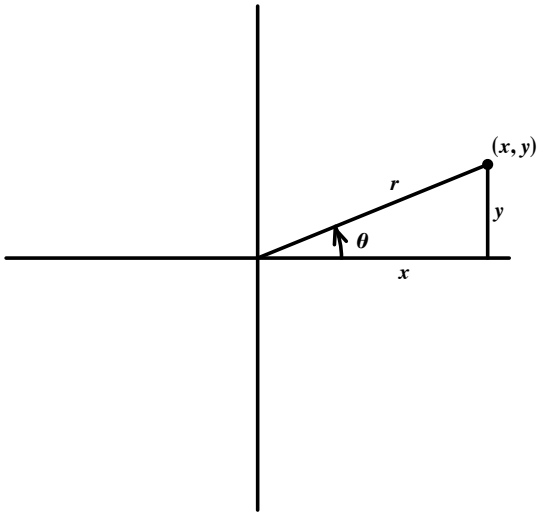
Name_____

More Special Angles (Trigonometry, Day 2)

In this handout we will continue using the circular trig definitions (x, y, and r, rather than adjacent, opposite, and hypotenuse).

Definitions: Circular Trigonometry

In order to evaluate trigonometric functions for angles of any size, we will now introduce a new set of definitions based on the idea we developed in the last problem. We begin all angles on the positive x-axis and rotate counterclockwise by θ. This establishes values for x, y, and r. We use these values to define our trig functions in the following way:



$\cos\theta = \frac{x}{r}$ $\qquad$ $\sec\theta = \frac{r}{x}$

$\sin\theta = \frac{y}{r}$ $\qquad$ $\csc\theta = \frac{r}{y}$

$\tan\theta = \frac{y}{x}$ $\qquad$ $\cot\theta = \frac{x}{y}$

Here are some important notes about our new definitions:

- We can choose any positive value for r. (Try to use whichever value is most convenient!)
- x and y could be either positive or negative, depending on which quadrant the point is in.
- We always start our angles on the positive x-axis.
- We rotate counterclockwise for positive angles and clockwise for negative angles. (Here's a way to think about this: Positive angles start off by going up and negative angles start off going down.

1. Find each of the following. Draw a diagram for each problem. Do not use a calculator. ***Note:*** Answers are provided at the end of this problem.

a. $\cos 150^\circ$

b. $\sin 30^\circ$

c. $\cos 45^\circ$

d. $\sin 225^\circ$

e. $\sin 240^\circ$

f. $\sin(-30^\circ)$

g. $\cos(-120^\circ)$

h. $\sin 135^\circ$

Answers

a. $-\frac{\sqrt{3}}{2}$ b. $\frac{1}{2}$ c. $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ d. $-\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ e. $-\frac{\sqrt{3}}{2}$ f. $-\frac{1}{2}$ g. $-\frac{1}{2}$ h. $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

2. Find each of the following. Draw a diagram for each problem. Do not use a calculator. ***Note:*** Answers are provided at the end of this problem.

a. $\tan 30^\circ$

b. $\tan 45^\circ$

c. $\tan 150^\circ$

d. $\tan(-60^\circ)$

Answers

a. $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ b. $\frac{1}{1} = 1$ c. $-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$ d. $-\frac{\sqrt{3}}{1} = -\sqrt{3}$

3. Find each of the following. Draw a diagram for each problem. Do not use a calculator. ***Note:*** Answers are provided at the end of this problem.

a. $\sin 90^\circ$

b. $\cos 90^\circ$

c. $\cos 180^\circ$

d. $\sin 0^\circ$

e. $\tan 180^\circ$

f. $\tan 90^\circ$

g. $\cos 0^\circ$

f. $\tan 0^\circ$

Answers

a. 1 b. 0 c. -1 d. 0 e. 0 f. Undefined g. 1 h. 0

4. Find each of the following. Draw a diagram for each problem. Do not use a calculator. ***Note:*** Answers are provided at the end of this problem.

a. $\sin 390^\circ$

b. $\cos 405^\circ$

c. $\sin 450^\circ$

d. $\tan 450^\circ$

e. $\sin 690^\circ$

f. $\tan 855^\circ$

Answers

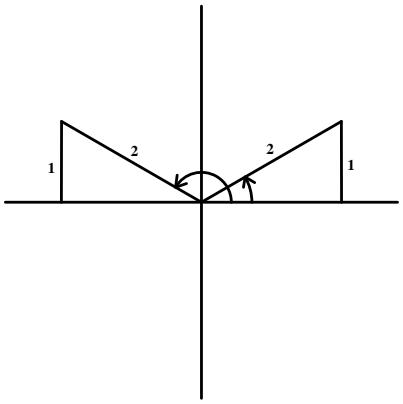
a. $\frac{1}{2}$ b. $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ c. 1 d. Undefined e. $-\frac{1}{2}$ f. -1

5. Consider the equation $\sin\theta = \frac{1}{2}$. Our goal is to find all solutions to this equation without using our calculators.

a. Which quadrants could θ be in? ***Hint:*** What do we know about the value of y? Is y positive or negative?

b. Since $\sin\theta = \frac{1}{2}$, we can conclude that y is positive. This means that θ must either be in quadrant 1 or quadrant 2. Make a diagram illustrating each of these possibilities. ***Note:*** Try to draw your diagram as accurately as possible!

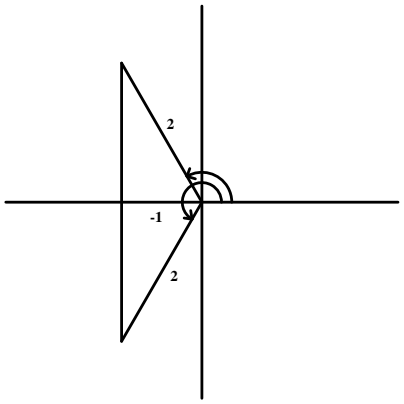
c. Here is the diagram you should have drawn in part **b**.



We can see that there are two possible values for θ. What are they?

d. You should have found that $\theta = 30^\circ$ or $\theta = 150^\circ$. Now use a similar approach to solve the equation $\cos\theta = -\frac{1}{2}$

Solution $\theta = 120^\circ$ or $\theta = 240^\circ$, as shown in the diagram below.



6. Solve each of the following equations for $0 \le \theta < 360^\circ$. Draw a diagram for each problem. Do not use a calculator. ***Note:*** Answers are provided at the end of this problem.

a. $\sin\theta = \frac{\sqrt{3}}{2}$

b. $\sin\theta = -\frac{\sqrt{2}}{2}$

c. $\cos\theta = \frac{\sqrt{3}}{2}$

d. $\cos\theta = -\frac{1}{2}$

e. $\tan\theta = \frac{1}{\sqrt{3}}$ ***Hint:*** $\frac{1}{\sqrt{3}} = \frac{-1}{-\sqrt{3}}$

f. $\tan\theta = 1$ ***Hint:*** $1 = \frac{1}{1} = \frac{-1}{-1}$

g. $\sin\theta = 1$

h. $\cos\theta = 0$

i. $\cos\theta = -1$

j. $\sin\theta = 0$

Answers

a. 60° or 120° b. 225° or 315° c. 30° or 330° d. 120° or 240° e. 30° or 210° f. 45° or 225° g. 90° h. 90° or 270° i. 180° h. 0° or 180°