Algebra 2/Pre-Calculus

Name_____

More Trigonometric Equations (Trigonometry, Day 8)

In this handout, we will practice a variety of equation solving techniques. All problems should be done **without the use of a calculator**.

1. Consider each of the following equations. Find all solutions.

a. Solve $\sin(\theta) = \frac{1}{2}$.

b. You should have found that $\theta = 30^\circ + 360^\circ N$ or $\theta = 150^\circ + 360^\circ N$.

Now solve the equation $\sin(2\theta) = \frac{1}{2}$.

c. Since $2\theta = 30^\circ + 360^\circ N$ or $2\theta = 150^\circ + 360^\circ N$, we can conclude that $\theta = 15^\circ + 180^\circ N$ or $\theta = 75^\circ + 180^\circ N$. (Notice that the dividing by 2 turns $360^\circ N$ into $180^\circ N$.)

Use a similar approach to solve the equation $\sin(5\theta - 10) = \frac{\sqrt{3}}{2}$.

d. Here's the solution to the last problem:

$$5\theta - 10 = 60^\circ + 360^\circ N \text{ or } 5\theta - 10 = 120^\circ + 360^\circ N$$

$$5\theta = 70^\circ + 360^\circ N \text{ or } 5\theta = 130^\circ + 360^\circ N$$

$$\theta = 14^\circ + 72^\circ N \text{ or } \theta = 26^\circ + 72^\circ N$$

Now solve the equation $\cos(2\theta + 8) = -\frac{1}{2}$.

e. You should have found that $\theta = 56^\circ + 180^\circ N$ or $\theta = 116^\circ + 180^\circ N$

Now solve the equation $\cos(\theta + 20) = \frac{\sqrt{2}}{2}$. ***Note:*** Answers are provided at the end of this problem.

f. Solve the equation $\tan(3\theta + 12) = \sqrt{3}$.

g. Solve the equation $\sin(2\theta + 14) = 0$.

h. Solve the equation $\tan(5\theta - 40) = -1$.

i. Solve the equation $\sin(2\theta) = 1$.

j. Solve the equation $\tan(\theta - 4) = 0$.

Some answers e. $\theta = 25^\circ + 360^\circ N$ or $\theta = 295^\circ + 360^\circ N$ f. $\theta = 16^\circ + 120^\circ N$ or $\theta = 76^\circ + 120^\circ N$
g. $\theta = -7^\circ + 180^\circ N$ or $\theta = 83^\circ + 180^\circ N$ h. $\theta = 35^\circ + 72^\circ N$ or $\theta = 71^\circ + 72^\circ N$
i. $\theta = 45^\circ + 180^\circ N$ j. $\theta = 4^\circ + 360^\circ N$ or $\theta = 184^\circ + 360^\circ N$

2. In this problem, we will explore different ways of writing the answer for a trigonometric equation.

a. Jermaine and Connor were both trying to solve the equation $\sin(\theta + 200) = \frac{1}{2}$.

Jermaine found that $\theta = -170^\circ + 360^\circ N$ or $\theta = -50^\circ + 360^\circ N$ whereas Connor found that $\theta = 190^\circ + 360^\circ N$ or $\theta = 310^\circ + 360^\circ N$. Who was right?

b. You should have found that Jermaine and Connor both had correct ways of writing the solution. What value of N would Jermaine need in order to get a solution of 190°? What value of N would Connor need in order to get a solution of 190°?

c. Explain why Connor's solutions and Jermaine's solutions actually give us the same values.

Answers a. They are both right. b. Jermaine: $N = 1$. Connor: $N = 0$. c. When the N-values in Jermaine's solutions are 1 larger than the N-values in Connor's solutions, they both get the same numbers.

3. In this problem, we will learn how to find solutions for a trigonometric equation on a given interval.

a. Solve $\sin(\theta) = \frac{1}{2}$. Find all solutions for $0^\circ \le \theta < 360^\circ$.

b. You should have found that $\theta = 30^\circ$ or $\theta = 150^\circ$. Now solve the equation $\sin(2\theta) = \frac{1}{2}$. Again, find all solutions for $0^\circ \le \theta < 360^\circ$.

c. Ted said that the equation in part **b** had two solutions: 15° or 75°, but Kaitlin said that it actually had four solutions: 15°, 75°, 195°, or 255°. Who is right? Explain.
Note: Remember, you can check each solution by plugging in back into the equation.

d. You should have found that all four of Kaitlin's solutions worked. But how did she find the solutions that Ted missed?

To answer this question, solve the equation $\sin(2\theta) = \frac{1}{2}$ again. This time, find all solutions (not just the ones on $0^\circ \le \theta < 360^\circ$).

e. You should have found that $\theta = 15^\circ + 180^\circ N$ or $\theta = 75^\circ + 180^\circ N$. What values do you get when $N = 0$? What values do you get when $N = 1$? When $N = 2$?

f. Here's what you should have found in the last problem:

$N = 0: \ \theta = 15^\circ$ or $\theta = 75^\circ$

$N = 1: \ \theta = 195^\circ$ or $\theta = 225^\circ$

$N = 2: \ \theta = 375^\circ$ or $\theta = 435^\circ$

Now let's return to the original problem: Explain how you can find all solutions for the equation $\sin(2\theta) = \frac{1}{2}$ on $0^\circ \le \theta < 360^\circ$.

4. The approach we used to solve the last problem is challenging, so we will practice it. Solve $\sin(3\theta - 18) = -\frac{\sqrt{3}}{2}$. Find all solutions for $0^\circ \le \theta < 360^\circ$. ***Note:*** The solution is provided below, so check your work after you complete the problem.

Solution: $\sin(3\theta - 18) = -\frac{\sqrt{3}}{2}$

$3\theta - 18 = 240^\circ + 360^\circ N$ or $3\theta - 18 = 300^\circ + 360^\circ N$

$3\theta = 258^\circ + 360^\circ N$ or $3\theta = 318^\circ + 360^\circ N$

$\theta = 86^\circ + 120^\circ N$ or $\theta = 106^\circ + 120^\circ N$

$N = 0: \ \theta = 86^\circ$ or $\theta = 106^\circ$

$N = 1: \ \theta = 206^\circ$ or $\theta = 226^\circ$

$N = 2: \ \theta = 326^\circ$ or $\theta = 346^\circ$

Thus, the equation has six solutions: 86°, 106°, 206°, 226°, 326°, 346°.

5. Solve each of the following equations. Find all solutions for $0^\circ \le \theta < 360^\circ$. ***Remember:*** No calculators!

a. $\cos(2\theta - 10) = \frac{1}{2}$

b. $\sin(3\theta) = \frac{\sqrt{2}}{2}$

c. $\tan(2\theta - 40) = \sqrt{3}$

d. $\cos(2\theta) = -1$

Answers a. $35^\circ, 155^\circ, 215^\circ, 335^\circ$ b. $15^\circ, 45^\circ, 135^\circ, 165^\circ, 255^\circ, 285^\circ$

c. $50^\circ, 140^\circ, 230^\circ, 320^\circ$ d. $90^\circ, 270^\circ$