Algebra 2/Pre-Calculus

Name_____

Pascal's Triangle and Probability (Day 5, Pascal's Triangle)

The goal of this handout is to explore the connections between Pascal's Triangle and certain probability problems.

1. Suppose you flipped a coin 3 times.

a. How many sequences of heads and tails are possible? (We consider HHT different from HTH because the heads and tails are in a different order.) ***Hint:*** How many possibilities are there for the first flip? For the second? For the third?

b. Hopefully, you found the answer to part **a** was 8. (2 possibilities for each flip, so $2 \cdot 2 \cdot 2 = 2^3 = 8$.) Here's a way of writing them:

HHH	HHT	HTT	TTT
	HTH	THT	
	THH	TTH	

How were they arranged? And what does this have to do with Pascal's Triangle?

c. Find each of the following:

$_3C_0 =$ $_3C_1 =$ $_3C_2 =$ $_3C_3 =$

d. How do these combination numbers relate to what's going on in part **b**? Why?

e. Find the value of $_3C_0 + {}_3C_1 + {}_3C_2 + {}_3C_3$. How does this relate to the problem with the coins?

2. Suppose you flipped a coin 4 times.

a. How many sequences of heads and tails are possible?

b. The answer to the last problem was 16 (2 possibilities for each flip, so $2 \cdot 2 \cdot 2 \cdot 2 = 2^4 = 16$.) Here's a way of writing them:

HHHH	HHHT	HHTT	TTTH	TTTT
	HHTH	HTHT	TTHT	
	HTHH	HTTH	THTT	
	THHH	THHT	HTTT	
		THTH		
		TTHH		

How were they arranged? And what does this have to do with Pascal's Triangle?

c. Find each of the following:

$_4C_0 =$ $_4C_1 =$ $_4C_2 =$ $_4C_3 =$ $_4C_4 =$

d. How do these combination numbers relate to what's going on in part **b**? Why?

e. Find the value of $_4C_0 + {}_4C_1 + {}_4C_2 + {}_4C_3 + {}_4C_4$. What's the idea?

3. Find each of the following.

a. $1 + 2 + 1 =$

b. $1 + 3 + 3 + 1 =$

c. $1 + 4 + 6 + 4 + 1 =$

d. $1 + 5 + 10 + 10 + 5 + 1 =$

e. Describe the pattern that you are seeing. If possible, try to explain why it works. (Consider using the coin flipping as part of your explanation.)

4. Suppose you flip a coin three times.

a. How many total sequences of are possible? (You've already answered this in an earlier question.)

b. How many of these have two heads and one tail (in any order)?

c. If you flip a coin three times, what is the probability that you will get two heads and one tail (in any order)?

5. Suppose you flip a coin four times.

a. What is the probability you will get three heads and one tail (in any order)?

b. What is the probability you will get two heads and two tails (in any order)?

c. What is the probability you will get all heads?

d. What is the probability you will get at least two heads?

6. Suppose you flip a coin five times.

a. What is the probability you will get exactly three heads?

b. What is the probability you will get exactly four heads?

c. What is the probability you will get either three or four heads?

d. What is the probability you will get less than two heads?

7. Suppose you flip a coin ten times.

a. What is the probability you will get exactly three heads?

b. What is the probability you will get exactly four heads?

c. What is the probability you will get at least eight heads?