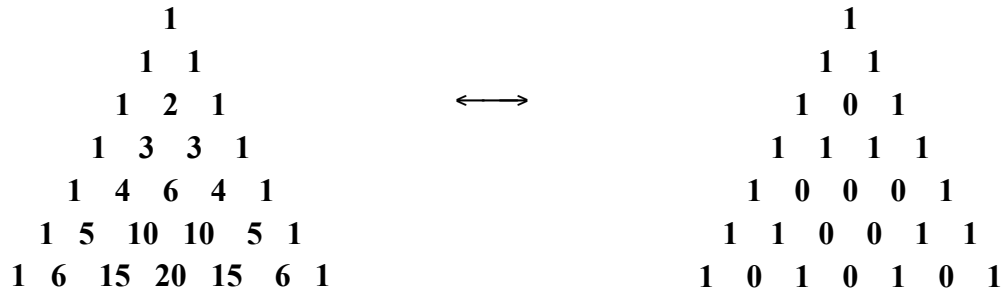**MATH 3**   Name_____
Pascal's Triangle, mod 2 (Day 7, Pascal's Triangle)
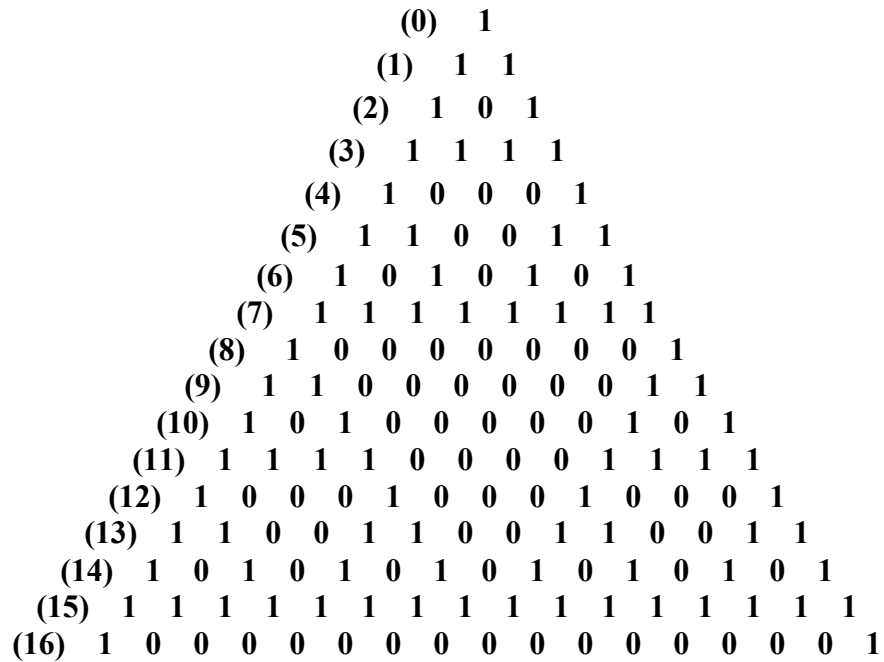
Pascal's Triangle with Ones and Zeros

1. Consider the two triangles below.

```
            1                                          1
          1   1                                      1   1
        1   2   1              ⟷                  1   0   1
      1   3   3   1                              1   1   1   1
    1   4   6   4   1                          1   0   0   0   1
  1   5  10  10   5   1                      1   1   0   0   1   1
1   6  15  20  15   6   1                  1   0   1   0   1   0   1
```
   Explain how the triangle on the right is related to Pascal's Triangle.

2. Continue the triangle on the right until the 16th row. Write tiny so that you will have enough space.

Here are the first 16 rows of the triangle from the previous page. This is called "Pascal's Triangle (mod 2)."

```
                          (0)   1
                       (1)   1   1
                      (2)   1  0  1
                    (3)   1  1  1  1
                   (4)   1  0  0  0  1
                  (5)   1  1  0  0  1  1
                (6)   1  0  1  0  1  0  1
               (7)   1  1  1  1  1  1  1  1
              (8)   1  0  0  0  0  0  0  0  1
            (9)   1  1  0  0  0  0  0  0  1  1
          (10)   1  0  1  0  0  0  0  0  1  0  1
        (11)   1  1  1  1  0  0  0  0  1  1  1  1
      (12)   1  0  0  0  1  0  0  0  1  0  0  0  1
     (13)   1  1  0  0  1  1  0  0  1  1  0  0  1  1
    (14)   1  0  1  0  1  0  1  0  1  0  1  0  1  0  1
   (15)   1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
  (16)   1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1
```

3. Take a moment to look over this triangle. Can you identify any patterns?

4. Which rows just have a 1 at the first and last spots?

5. Which rows are all 1's?

6. Which rows alternate 1's and 0's?

7. How many 1's will there be on the 32nd row? On the 64th row? What is the pattern?
   Hint: How many 1's were there on the 2nd row? The 4th row? The 8th row? The 16th row?

8. How many 1's will there be on the 31th row? On the 63rd row? What is the pattern?
   Hint: How are these numbers related to the numbers in the last problem?

9. Complete the following table relating the row number to the number of 1's on that row.

Row	# of 1's		Row	# of 1's
1	2		9	
2	2		10	
3	4		11	
4	2		12	
5			13	
6			14	
7			15	
8			16	

10. What do you notice about these numbers? (There are lots of possible answers to this question!)

11. Look back at Pascal's triangle (the usual version, not just with 0's and 1's.) How many odd numbers are there on the 4th row of Pascal's Triangle? How many odd numbers on the 7th row of Pascal's Triangle? How many odd numbers on the 14th row of Pascal's Triangle? *Hint:* Think about the connection between the original Pascal's Triangle and Pascal's Triangle (mod 2).

12. How many odd numbers are there on the 64th row of Pascal's Triangle? How many even numbers are on this row? What about the 63rd row? The 65th row?

13. Optional Challenge Can you continue the table from the last page without continuing to make the triangle?

Row	# of 1's
17	2
18	2
19	4
20	2
21	
22	
23	
24	

Row	# of 1's
25	
26	
27	
28	
29	
30	
31	
32	

14. Optional Challenge How many odd numbers are there on the 100th row of Pascal's Triangle? *Note:* This is a repeat of the challenge problem that you were given at the end of the last packet. Even though this problem is about the usual Pascal Triangle, it's helpful to think about the connections to Pascal's Triangle (mod 2). But even with this hint, it's still a pretty hard problem!
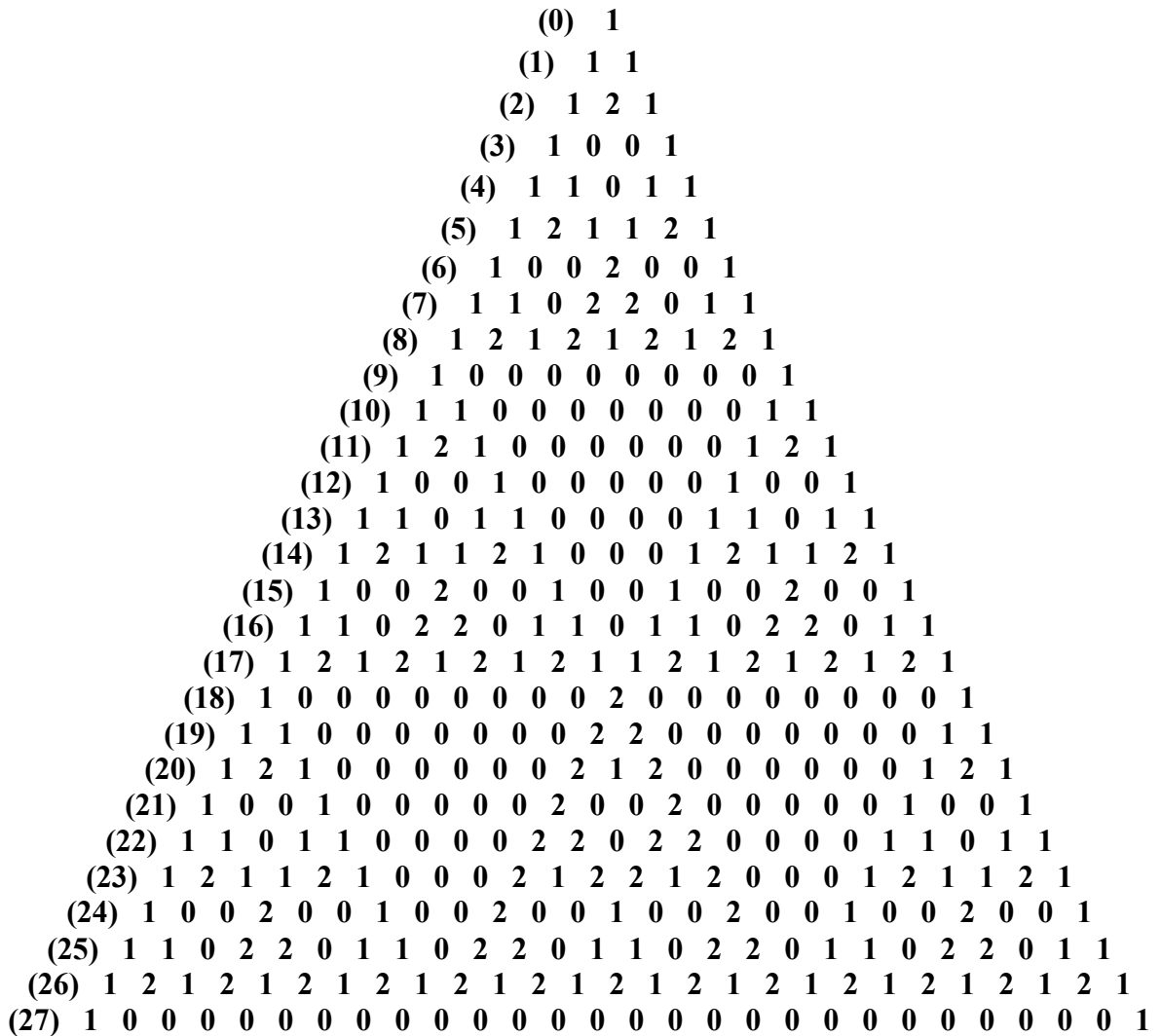
Pascal's Triangle with 0, 1, and 2

In the last group of problems, we considered Pascal's Triangle (mod 2). We used a special set of addition rules. Specifically, we pretended that $1+1=0$.

Now we will explore Pascal's Triangle (mod 3). Again, we will have some unusual rules for addition: $1+2=0$ and $2+2=1$. We will also have some familiar rules: $0+0=0$, $0+1=1$, and $1+1=2$. With these rules in mind, fill in the next few rows. Try to write small enough to make it to the 18th row!

$$
\begin{array}{c}
\textbf{(0)} \quad 1 \\
\textbf{(1)} \quad 1 \quad 1 \\
\textbf{(2)} \quad 1 \quad 2 \quad 1 \\
\textbf{(3)} \quad 1 \quad 0 \quad 0 \quad 1 \\
\textbf{(4)} \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \\
\textbf{(5)} \quad 1 \quad 2 \quad 1 \quad 1 \quad 2 \quad 1 \\
\textbf{(6)} \quad 1 \quad 0 \quad 0 \quad 2 \quad 0 \quad 0 \quad 1 \\
\textbf{(7)} \quad 1 \quad 1 \quad 0 \quad 2 \quad 2 \quad 0 \quad 1 \quad 1
\end{array}
$$

Here are the first 27 rows of Pascal's Triangle (mod 3). Take a moment to compare this to your answer for the last problem.

```
                              (0)   1
                             (1)   1  1
                            (2)   1  2  1
                           (3)   1  0  0  1
                          (4)   1  1  0  1  1
                         (5)   1  2  1  1  2  1
                        (6)   1  0  0  2  0  0  1
                       (7)   1  1  0  2  2  0  1  1
                      (8)   1  2  1  2  1  2  1  2  1
                     (9)   1  0  0  0  0  0  0  0  0  1
                    (10)  1  1  0  0  0  0  0  0  0  1  1
                   (11)  1  2  1  0  0  0  0  0  0  1  2  1
                  (12)  1  0  0  1  0  0  0  0  0  1  0  0  1
                 (13)  1  1  0  1  1  0  0  0  0  1  1  0  1  1
                (14)  1  2  1  1  2  1  0  0  0  1  2  1  1  2  1
               (15)  1  0  0  2  0  0  1  0  0  1  0  0  2  0  0  1
              (16)  1  1  0  2  2  0  1  1  0  1  1  0  2  2  0  1  1
             (17)  1  2  1  2  1  2  1  2  1  1  2  1  2  1  2  1  2  1
            (18)  1  0  0  0  0  0  0  0  0  2  0  0  0  0  0  0  0  0  1
           (19)  1  1  0  0  0  0  0  0  0  2  2  0  0  0  0  0  0  0  1  1
          (20)  1  2  1  0  0  0  0  0  0  2  1  2  0  0  0  0  0  0  1  2  1
         (21)  1  0  0  1  0  0  0  0  0  2  0  0  2  0  0  0  0  0  1  0  0  1
        (22)  1  1  0  1  1  0  0  0  0  2  2  0  2  2  0  0  0  0  1  1  0  1  1
       (23)  1  2  1  1  2  1  0  0  0  2  1  2  2  1  2  0  0  0  1  2  1  1  2  1
      (24)  1  0  0  2  0  0  1  0  0  2  0  0  2  0  0  1  0  0  2  0  0  1  0  0  1
     (25)  1  1  0  2  2  0  1  1  0  2  2  0  2  2  0  1  1  0  2  2  0  1  1  0  1  1
    (26)  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1  2  1
   (27)  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1
```

15. Take a moment to look over this triangle. What do you notice? How is it similar to Pascal's Triangle (mod 2)? How is it different? *Note:* There are many ways to answer this question!

16. Do the 0's in Pascal's Triangle (mod 3) appear to form triangles? Is this similar or different from Pascal's Triangle (mod 2)? Explain.

17. Which rows of Pascal's Triangle (mod 3) are all zeros except for the 1's in the first and last spots? Explain the pattern.

18. Which rows of Pascal's Triangle (mod 3) alternate 1's and 2's (but have no zeros). Explain.

19. How many zeros are there on the 81st row of Pascal's Triangle (mod 3)? Explain how you got your answer.

20. How many zeros are there on the 80th row of Pascal's Triangle (mod 3)? Explain how you got your answer.

21. Look back at the usual Pascal's Triangle. How many numbers on the 6th row of Pascal's Triangle are divisible by 3? How many numbers on the 7th row of Pascal's Triangle are divisible by 3? The 8th row? The 9th row? ***Hint:*** How is this related to Pascal's Triangle (mod 3)?

22. Look back at the usual Pascal's Triangle. How many numbers on the 25th row of Pascal's Triangle are divisible by 3? How many on the 26th row? How many on the 27th row? Explain how you got your answer. ***Note:*** Don't actually write out the first 27 rows of Pascal's Triangle! Instead, use Pascal's Triangle (mod 3) to get the answers.

23. Optional Challenge Problem The 5th row of Pascal's Triangle is 1 5 10 10 5 1 and the 7th row of Pascal's Triangle is 1 7 21 35 35 21 7 1. Notice that all of the numbers on the 5th row are divisible by 5 and all of the numbers on the 7th row are divisible by 7 (aside from the 1's on the two ends). Are there any other rows that have this property? Which ones? Explain.