Algebra 2/Pre-Calculus

Name_____

Properties of Means and Variance (Day 4, Statistics)

In this handout, we will investigate what happens when we add or multiply means and variance. We begin by reviewing our definitions of mean, variance, and standard deviation.

Mean: $\bar{x} = \frac{1}{n}(x_1 + x_2 + ... + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i$

Variance: $\sigma^2 = \frac{1}{n}\left((x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + ... + (x_n - \bar{x})^2\right) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

Standard deviation: $\sigma = \sqrt{\frac{1}{n}\left((x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + ... + (x_n - \bar{x})^2\right)} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$

1. Suppose you have the following two spinners.

Spinner A: {2, 2, 2, 2, 4, 4, 8, 8} **Spinner B:** {5, 5, 5, 5, 7, 7, 11, 11}

a. How are the numbers on spinner B related to the numbers on spinner A?

b. Calculate the mean for Spinner A and the mean for Spinner B.

c. How are the means for the two spinners related?

d. You should have found that the mean for Spinner A was 4 and the mean for Spinner B was 7, so the mean for Spinner B was 3 more than the mean for Spinner A. Will this type of relationship always happen in this sort of situation? Explain.

1. **(Continued)** Here's a reminder of the spinners. (Same as on the last page.)

   Spinner A: {2, 2, 2, 2, 4, 4, 8, 8} **Spinner B:** {5, 5, 5, 5, 7, 7, 11, 11}

   e. Calculate the variance and standard deviation for Spinner A.

   f. You should have found that the variance was 6 and the standard deviation was 2.4494897. (If you didn't get these numbers, go back and check your work.) Now calculate the variance and standard deviation for Spinner B.

   g. You should have gotten the same answers in parts e and f. Why did this happen?

2. Suppose you have the following two spinners. ***Note:*** Spinner A is the same as the last problem.

Spinner A: {2, 2, 2, 2, 4, 4, 8, 8}

Spinner C: {10, 10, 10, 10, 20, 20, 40, 40}

a. How are the numbers on spinner C related to the numbers on spinner A?

b. Calculate the mean for Spinner C.

c. How are the means for the two spinners related?

d. You should have found that the mean for Spinner C was 5 times the mean for Spinner A. Is this a coincidence? Will it always happen this way? Why does this make sense?

2. (Continued) Here's a reminder of the spinners. (Same as on the last page.)

Spinner A: {2, 2, 2, 2, 4, 4, 8, 8}

Spinner C: {10, 10, 10, 10, 20, 20, 40, 40}

e. Calculate the variance and standard deviation for Spinner C.

f. How is the standard deviation for Spinner C related to the standard deviation for Spinner A? ***Hint:*** The numbers on Spinner C are 5 times bigger than the numbers on Spinner A.

g. You should have found that the standard deviation for Spinner C was five time the size of the standard deviation for Spinner A. Is the variance for Spinner C five times the variance for Spinner A? Is there some other relationship?

3. Suppose you have the following two spinners. ***Note:*** Spinner A is the same as the last problem two problems.

Spinner A: {2, 2, 2, 2, 4, 4, 8, 8} **Spinner D:** {1, 3, 5}

a. Calculate the mean, the variance, and the standard deviation for Spinner D.

b. Suppose you spin both of the spinners and find the sum. Write the polynomial multiplication associated this situation. Then find the product. (You may use wolframalpha.)

c. Find the mean for the sum. ***Note:*** There are 8 equally likely options for Spinner A and 3 equally likely options for Spinner D, so there are 24 equally likely options for the sum.

d. How did the mean of the sum relate to the mean for each spinner? There should be an obvious relationship.

3. (Continued) Here's a reminder of the spinners. (Same as on the last page.)
Spinner A: {2, 2, 2, 2, 4, 4, 8, 8} **Spinner D:** {1, 3, 5}

e. Calculate the variance and standard deviation for the sum. Note: This calculation will take a bit of time. Use the ideas from the first problem to speed it up.

f. How are the variances for Spinner A and Spinner D related to the variance for the sum? There should be an obvious relationship.

g. You should have found that the variance of the sum was equal to the sum of the variances for Spinners A and D. (Double check your work if you didn't get that.). Does it work the same way for the standard deviation? In other words, is the standard deviation of the sum equal to the sum of the standard deviations for the two spinners?

4. Suppose you have the following two spinners.

Spinner E: {3, 3, 5, 5, 5, 6, 6, 6, 8, 8, 9} **Spinner F:** {2, 2, 2, 5, 5, 5}

a. Find the mean, variance, and standard deviation for each spinner.

b. Suppose both Spinners are spun at the same time. What will be the mean, variance and standard deviation for the sum? ***Hint:*** Don't do this calculation from scratch! Use your answers from part **a**.

c. Suppose Spinner E is spun five times. Find the mean, variance, and standard deviation for the sum of the five spins. ***Hint:*** Don't do this calculation from scratch! Use the information that you already know about spinner E.

d. Suppose Spinner E is spun 3 times and Spinner F is spun 4 times. Find the mean, variance, and standard deviation for the sum of all 7 spins.

e. Suppose Spinner G and Spinner H are as follows:

Spinner G: {9, 9, 15, 15, 15, 18, 18, 18, 24, 24, 27}

Spinner H: {8, 8, 8, 20, 20, 20}

How are these Spinners related to Spinners E and F?

f. Find the mean, variance, and standard deviation for the sum of Spinners G and H.

5. Suppose you are flipping a fair coin and counting the number of times it lasts heads.

a. Explain why this coin flip is equivalent to the Spinner {0, 1}.

b. Find the mean, variance, and standard deviation for the number of heads from one flip of the coin.

c. You should have found that the mean was 0.5, the variance was 0.25, and the standard deviation was 0.5. Suppose we flip the coin 100 times. What are the mean, variance, and standard deviation for the number of heads from 100 flips?

d. You should have found that the mean was 50, the variance was 25, and the standard deviation was 5. What are the mean, variance, and standard deviation for the number of heads from n flips?