Algebra 2/Pre-Calculus

Name_____

Pythagorean Identity and More Practice (Trigonometry, Day 4)

In this handout we will continue using the circular trig definitions (x, y, and r, rather than adjacent, opposite, and hypotenuse). All of the problems on this handout should be done **without the aid of a calculator**.

1. Find each of the following. Do not use a calculator.

a. Find $\cos 30^\circ$, $\sin 30^\circ$, and $(\cos 30^\circ)^2 + (\sin 30^\circ)^2$.

b. Find $\cos 45^\circ$, $\sin 45^\circ$, and $(\cos 45^\circ)^2 + (\sin 45^\circ)^2$.

c. Find $\cos 60^\circ$, $\sin 60^\circ$, and $(\cos 60^\circ)^2 + (\sin 60^\circ)^2$.

d. Find $\cos 90^\circ$, $\sin 90^\circ$, and $(\cos 90^\circ)^2 + (\sin 90^\circ)^2$.

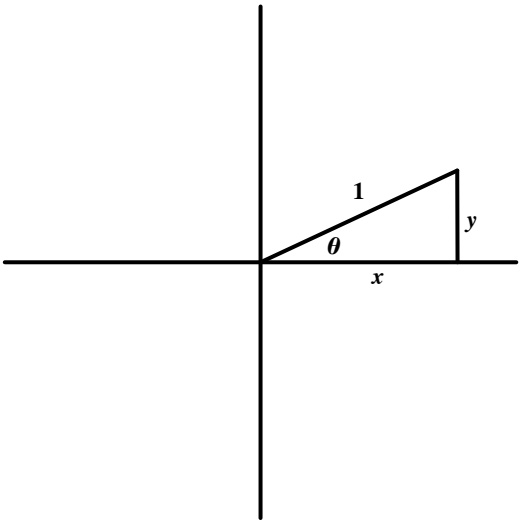
e. Find $\cos 120^\circ$, $\sin 120^\circ$, and $(\cos 120^\circ)^2 + (\sin 120^\circ)^2$.

f. Find $\cos 180^\circ$, $\sin 180^\circ$, and $(\cos 180^\circ)^2 + (\sin 180^\circ)^2$.

2. Look back at problem **1**. Make a conjecture regarding the value of $(\cos\theta)^2 + (\sin\theta)^2$. Can you explain why this is true?

3. In the last problem, you should have found that $(\cos\theta)^2 + (\sin\theta)^2 = 1$. This is often called a Pythagorean identity. Can you explain why?

4. Prove that $(\cos\theta)^2 + (\sin\theta)^2 = 1$. ***Note:*** A helpful diagram is provided below.



Solution First, we observe that $\cos\theta = \frac{x}{1} = x$ and $\sin\theta = \frac{y}{1} = y$, so
$(\cos\theta)^2 + (\sin\theta)^2 = x^2 + y^2 = 1$

5. Suppose $\cos\theta = 0.8$. What is the value of $\sin\theta$? ***Hint:*** There are two possible answers.

Answer $\sin\theta = 0.6$ or $\sin\theta = -0.6$

6. Suppose $\sin\theta = -0.7$. What is the value of $\cos\theta$? ***Hint:*** There are two possible answers, both of which will involve square roots.

Answer $\cos\theta = \sqrt{0.51}$ or $\cos\theta = -\sqrt{0.51}$

7. Suppose $\cos\theta = 0.6$. What are the possible values of $\tan\theta$?

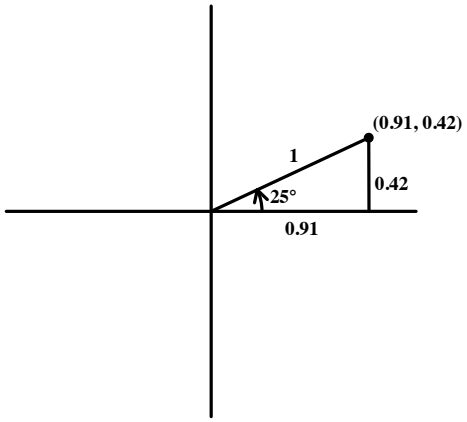
Answer $\tan\theta = \frac{0.8}{0.6} = \frac{8}{6} = \frac{4}{3}$ or $\tan\theta = \frac{-0.8}{0.6} = \frac{-8}{6} = \frac{-4}{3}$

8. The value of $\cos 25^\circ$ is approximately 0.91.

a. **Without** using the trig functions on your calculator, find the value of $\sin 25^\circ$.

b. Draw a diagram for a 25° on a graph. Include the values for x and y. ***Hint:*** Make $r = 1$.

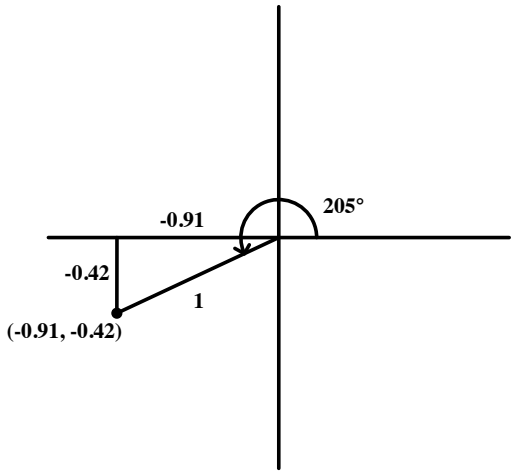
c. Here's the diagram you should have drawn in part b.



Now find the value of $\tan 25^\circ$ without using your calculator.

d. Now draw a diagram for a 205° on a graph. Include the values for x and y. Again, make $r = 1$. ***Hint:*** $205 = 25 + 180$.

e. Here's the diagram you should have drawn in part **d**.



Without using a calculator, find the value of $\cos 205^\circ$. ***Hint:*** Use the diagram you made in part **d**.

f. Without using a calculator, find the value of $\sin 205^\circ$.

g. Without using a calculator, find the value of $\cos 155^\circ$ and $\sin 155^\circ$. Begin by drawing a diagram for 155°. ***Hint:*** $155 = 180 - 25$.

h. Without using a calculator, find the value of $\cos 115^\circ$ and $\sin 115^\circ$.

Hint: $115 = 25 + 90$.

i. Without using a calculator, find the value of $\cos 65^\circ$ and $\sin 65^\circ$.

Hint: $65 = 90 - 25$.

Some answers a. 0.42 c. 0.47 e. -0.91 f. -0.42 g. $\cos 155^\circ = -0.91$ and $\sin 155^\circ = 0.42$ h. $\cos 115^\circ = -0.42$ and $\sin 115^\circ = 0.91$ i. $\cos 65^\circ = 0.42$ and $\sin 65^\circ = 0.91$

9. Solve each of the following equations **without** using your calculator. Find all solutions on $0^\circ \le \theta < 360^\circ$

a. $\sin\theta = \frac{\sqrt{3}}{2}$

b. $\cos\theta = \frac{\sqrt{2}}{2}$

c. $\sin\theta = 1$

d. $\sin\theta = 0$

e. $\cos\theta = -\frac{\sqrt{3}}{2}$

f. $\cos\theta = -1$

g. $\tan\theta = 1$

h. $\tan\theta = \sqrt{3}$

i. $\tan\theta = -\sqrt{3}$

j. $\tan\theta = 0$

Answers a. 60 or 120 b. 45 or 315 c. 90 d. 0 or 180 e. 150 or 210 f. 180 g. 45 or 225 h. 60 or 240 i. 120 or 300 j. 0 or 180