## Algebra 2/Pre-Calculus

Name_____

Circular Trig Review

Carefully complete each of the following problems. Show work wherever appropriate. Do all problems without the aid of a calculator.

**1.** Find each of the following without a calculator. Draw a diagram for each problem, but do it quickly! These should be easy.

**a.** $\sin 225^\circ$

**b.** $\cos 0^\circ$

**c.** $\cos(-150^\circ)$

**d.** $\tan 330^\circ$

**e.** $\sin 630^\circ$

**f.** $\cos 90^\circ$

**g.** $\tan 180^\circ$

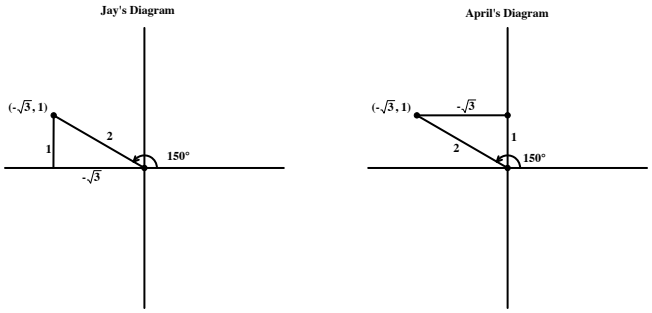
**h.** $\tan 90^\circ$

**i.** $\tan 225^\circ$

**j.** $\tan(-120^\circ)$

**Answers** a. $-\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ b. 1 c. $-\frac{\sqrt{3}}{2}$ d. $-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$ e. -1 f. 0 g. 0 h. undefined i. 1 j. $\sqrt{3}$

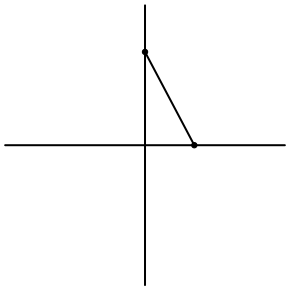
**2.** April and Jay were trying to find $\sin 150^\circ$. The diagrams they drew are presented below.




Whose diagram is correct? Explain.

**Answer** Both diagrams are fine because both result in the same terminal point at $(-\sqrt{3},1)$.

**3.** Another student in their class was trying to find the value of $\sin 90^\circ$. Here is the beginning of that student's diagram.



Is this a good way to start this problem?

**Answer** It is unlikely that this diagram will lead anywhere good.

**4.** Solve each of the following equations. Find all solutions. ***Reminder:*** Show all work and include a diagram for each problem.

**a.** $2\sin\theta + 5 = 6$

**b.** $\cos\theta = -1$

**c.** $\tan(2\theta + 7) = 1$

**d.** $\sin(4\theta) = 0$

**Answers** a. $\theta = 30^\circ + 360^\circ N$ or $\theta = 150^\circ + 360^\circ N$ b. $\theta = 180^\circ + 360^\circ N$

c. $\theta = 19^\circ + 180^\circ N$ or $\theta = 109^\circ + 180^\circ N$ d. $\theta = 0^\circ + 90^\circ N$ or $\theta = 45^\circ + 90^\circ N$

**e.** $\cos^2\theta - \cos\theta = 0$

**f.** $2\sin^2\theta + 1 = 3\sin\theta$

**g.** $4\cos^2\theta = 1$

**h.** $\tan^2\theta = 3$

**Answers** e. $\theta = 0^\circ + 360^\circ N$ or $\theta = 90^\circ + 360^\circ N$ or $\theta = 270^\circ + 360^\circ N$

f. $\theta = 30^\circ + 360^\circ N$ or $\theta = 90^\circ + 360^\circ N$ or $\theta = 150^\circ + 360^\circ N$

g. $\theta = 60^\circ + 360^\circ N$ or $\theta = 120^\circ + 360^\circ N$ or $\theta = 240^\circ + 360^\circ N$ or $\theta = 300^\circ + 360^\circ N$

h. $\theta = 60^\circ + 360^\circ N$ or $\theta = 120^\circ + 360^\circ N$ or $\theta = 240^\circ + 360^\circ N$ or $\theta = 300^\circ + 360^\circ N$

**5.** In the last problem, you should have found that the equations $4\cos^2\theta = 1$ and $\tan^2\theta = 3$ had exactly the same solutions. Explain why this happened.

**6.** Jinyung and Jennifer were trying to solve the equation $\tan\theta = \frac{1}{\sqrt{3}}$. Jinyung said that solutions were $\theta = 30^\circ + 360^\circ N$ or $\theta = 210^\circ + 360^\circ N$ whereas Jennifer thought the solution was simply $\theta = 30^\circ + 180^\circ N$. Who was right? Explain.

**Answer** They are both right. At first, it looks like Jennifer has missed half the solutions, but in her solution, if $N = 1$, then $\theta = 210^\circ$. No solutions are missed.

**7.** Giulia and Akshay were trying to solve the equation $\sin\theta = \frac{\sqrt{3}}{2}$. Giulia said that solutions were $\theta = 60^\circ + 360^\circ N$ or $\theta = 120^\circ + 360^\circ N$ whereas Akshay thought the solution was simply $\theta = 60^\circ + 180^\circ N$. Who was right? Explain.

**Answer** Giulia's solution is correct, but Akshay's is not. In Akshay's solution, if $N = 1$, then $\theta = 240^\circ$. But $\theta = 240^\circ$ is not a solution to this equation.

**8.** Solve each of the following equations. Find all solutions for $0^\circ \le \theta < 360^\circ$. Remember to show all work and include a diagram for each problem. ***Caution:*** Check your answer after solving each equation to make sure you have found all of the solutions!

**a.** $\cos(2\theta) = \frac{\sqrt{2}}{2}$

**b.** $2\sin(3\theta + 6) + 4 = 3$

**c.** $\tan(\theta - 50) = 1$

**d.** $\sin(3\theta) = 1$

**Answers** a. 22.5, 157.5, 202.5, 337.5 b. 68, 108, 188, 228, 308, 348 c. 95, 275 d. 30, 150, 270

**9.** Solve each of the following. Find all solutions such that $0^\circ \le \theta < 360^\circ$. ***Note:*** Do not use your calculator on this problem.

**a.** $\sin^2\theta - \sin\theta = 0$

**b.** $2\cos^2\theta = \cos\theta$

**c.** $2\sin^2\theta + 1 = 3\sin\theta$

**d.** $2\cos^2\theta = \sqrt{3}\cos\theta$

**e.** $\tan^2\theta = \tan\theta$

**f.** $2\sin^3\theta = \sin^2\theta + \sin\theta$

**g.** $1 - \cos\theta = \sin^2\theta$

**h.** $3\cos\theta + 3 = 2\sin^2\theta$

**i.** $\sin^2\theta = \frac{1}{4}$

**Answers** a. 0, 90, 180 b. 60, 90, 270, 300 c. 30, 90, 150 d. 30, 90, 270, 330

e. 0, 45, 180, 225 f. 0, 90, 180, 210, 330 g. 0, 90, 270 h. 120, 180, 240

i. 30, 150, 210, 330

**10.** Determine whether each of the following expressions is equal to $\sin\theta$, $\cos\theta$, $-\sin\theta$, or $-\cos\theta$. Draw a diagram for each part, just as you did in the previous problems. Make sure it is clear how you got your answer. ***Note:*** Answers are provided at the end of this problem.

**a.** $\sin(180-\theta)$

**b.** $\cos(180-\theta)$

**c.** $\sin(\theta + 90)$

**d.** $\cos(\theta + 90)$

**e.** $\sin(-\theta)$

**f.** $\cos(-\theta)$

**Answers** a. $\sin\theta$ b. $-\cos\theta$ c. $\cos\theta$ d. $-\sin\theta$ e. $-\sin\theta$ f. $\cos\theta$