Algebra 2/Pre-Calculus **Name**_____

Extra Problems (Review of Days 1 – 9)

1. The goal of this problem is to prove the Bernoulli formulas:

Mean = np, Variance = $np(1-p)$, Standard deviation = $\sqrt{np(1-p)}$

To prove these formulas, we will imagine a bent coin that lands heads with a probability of p. We will first find the mean, variance, and standard deviation for a single flip of this coin, and then for *n* flips.

a. Find the mean for the number of heads from a single flip of the coin.

b. Find the variance for the number of heads from a single flip of the coin.

c. Now suppose we flip the coin *n* times and count the number of heads. Explain why the mean is np and the variance is $np(1-p)$.

d. Explain why the standard deviation for the number of heads from *n* flips of the coin is $\sqrt{np(1-p)}$.

Answers a. $p(1)+(1-p)(0)=p$ b. $p(1-p)^2+(1-p)(0-p)^2=p(1-2p+p^2)+(1-p)p^2$

$=p-2p^2+p^3+p^2-p^3=p-p^2=p(1-p)$ c. To find the mean and the variance for a repeated experiment, we simply multiply by the number of times it is repeated. d. To find the standard deviation, we take the square root of the variance.

2. Consider the following spinners.

Spinner A: {2, 2, 2, 4, 4} **Spinner B:** {1, 1, 2, 2, 5, 5}

a. Write a polynomial representing each spinner. Then find the product of the polynomials.

b. Suppose both spinners are spun. What is the probability that the sum of the numbers on the spinners will be a 5?

c. Suppose both spinners are spun. What is the probability that the sum of the numbers on the spinners will be greater than 5?

d. What is the expected value for Spinner A? What is the expected value for Spinner B?

e. Find the variance and the standard deviation each spinner.

f. Find the mean, variance, and standard deviation for the sum of the two spinners.

Answers a. $(3x^2 + 2x^4)(2x^1 + 2x^2 + 2x^5) = 6x^3 + 6x^4 + 4x^5 + 4x^6 + 6x^7 + 4x^9$

b. $\frac{4}{30}$ c. $\frac{14}{30}$ d. Spinner A: 2.8, Spinner B: 2.6667 (Remember: "Expected value" is just a synonym for the mean.) e. Spinner A: Variance = 0.96 and standard deviation = 0.9798, Spinner B: Variance = 2.8888 and standard deviation = 1.700.

f. Mean = $2.8 + 2.6667 = 5.4667$, variance = $0.96 + 2.8888 = 3.8488$, standard deviation = 1.9618.

3. Suppose you are taking a multiple choice test with 24 questions. Each question has four possible answers (A, B, C, D). You are completely unprepared for the test, so you guess on every question.

a. Find the mean, variance, and standard deviation for the number of questions you get right. ***Hint:*** What is your probability of getting a right answer on any one question?

b. Which values are within one standard deviation of the mean?

c. What is the probability of getting exactly 6 questions right? (You may use your calculator, but do not use wolframalpha.)

d. What is the probability of getting exactly 12 questions right? (You may use your calculator, but do not use wolframalpha.)

e. Use wolframalpha to find the probability that the number of wins is within 1 standard deviation of the mean. ***Hint:*** What expression should you put into wolframalpha?

Answers a. mean = $24(\frac{1}{4}) = 6$, variance = $24(\frac{1}{4})(\frac{3}{4}) = 4.5$, standard deviation = 2.1213 b. 4, 5, 6, 7, 8 c. $({}_{24}C_6)(\frac{1}{4})^6(\frac{3}{4})^{18} = 0.1853$ d. $({}_{24}C_{12})(\frac{1}{4})^{12}(\frac{3}{4})^{12} = 0.0051$

e. Each question is right or wrong, so input $(.25r + .75w)^{24}$. Then find the probability using the coefficients: $.1316 + .1755 + .1853 + .1588 + .1125 = .7637$.

4. A six-sided die is weighted so that the probability of getting each number is not the same. (The probabilities are given on the table below.)

a. Find the mean, variance, and standard deviation for a single roll of this die.

Number	***Probability***
1	0.05
2	0.1
3	0.1
4	0.2
5	0.25
6	0.3

b. Suppose this die is rolled five times. Find the mean, variance, and standard deviation for the sum of the five rolls.

c. Suppose this die is rolled twice. What is the probability that the sum will be 12?

Answers a. mean = $0.05(1)+0.1(2)+0.1(3)+0.2(4)+0.25(5)+0.3(6)=4.4$,variance= $0.05(1-4.4)^2+0.1(2-4.4)^2+0.1(3-4.4)^2+0.2(4-4.4)^2+0.25(5-4.4)^2+0.3(6-4.4)^2=2.24$, standard deviation = 1.4967 b. mean = $4.4(5)=22$, variance = $2.24(5)=11.2$, standard deviation = 3.2467 c. $(0.3)(0.3)=0.09$

5. There are 25 students in Mr. Verner's Math 3 class. They recently took a quiz that was graded out of 10 points. Their scores are listed on the table below

a. Find the mean, variance, and standard deviation for the quiz score.

Score	***Frequency***
5	2
6	1
7	0
8	5
9	11
10	6

Answers $\frac{1}{25}(2(5)+1(6)+0(7)+5(8)+11(9)+6(10))=8.6$ variance =

$\frac{1}{25}(2(5-8.6)^2+1(6-8.6^2)+5(8-8.6)^2+11(9-8.6)^2+6(10-8.6)^2)=8.6$

6. Suppose you flip a fair coin 8 times. Complete the following table.

# of heads	# of ways	probability
0		
1		
2		
3		
4		
5		
6		
7		
8		

Answer

# of heads	# of ways	probability
0	1	1/256
1	8	8/256
2	28	28/256
3	56	56/256
4	70	70/256
5	56	56/256
6	28	28/256
7	8	8/256
8	1	1/256

7. A bent coin lands heads 62% of the time. The coin is flipped five times. Complete the following table. Then sketch a histogram.

# of heads	# of ways	probability
0		
1		
2		
3		
4		
5		

Answer

# of heads	# of ways	probability
0	1	0.0079
1	5	0.0646
2	10	0.2110
3	10	0.3441
4	5	0.2808
5	1	0.0916

8. Oliver and Nicole are playing a game that involves rolling two six sided dice. Oliver says, "Let's just roll one six-sided dice and double the result. That'll do the same thing as rolling two six sided die and finding the sum."

Is rolling one six-sided die and doubling the result equivalent to rolling two six-sided dice and taking the sum? Explain. ***Note:*** Your explanation should address ideas about the mean, the variance, and the standard deviation.

Answer These are **not** the same. Both approaches result in an expected value of 7, but the variance and standard deviation are different, as shown below:

One die: mean = 3.5, variance = 2.9167, standard deviation = 1.7078

Two dice: mean = $3.5(2) = 7$, variance = $2.9167(2) = 5.8334$,

standard deviation = $\sqrt{5.8334} = 2.4152$

One die (doubled): mean = $3.5(2) = 7$, standard deviation = $1.7078(2) = 3.4156$,

Variance = $3.4156^2 = 11.6663$