In this problem set, we will learn to solve equations involving rational expressions.

1. Consider the following equation: $\dfrac{3}{7}x + 1 = \dfrac{31}{7}$

    a. Show how you can eliminate the fractions from this equation by multiplying both sides by the same number. Then solve the equation.

    b. Here's how you could have solved the last equation:

$$\frac{3}{7}x + 1 = \frac{31}{7}$$

$$\frac{3}{7}x + \frac{7}{7} = \frac{31}{7}$$

$$7\left(\frac{3}{7}x + \frac{7}{7}\right) = \left(\frac{31}{7}\right)7$$

$$3x + 7 = 31$$

$$3x = 24$$

$$x = 8$$

Now use the same type of strategy to solve the following equation: $\dfrac{2}{3}x + \dfrac{5}{2} = \dfrac{8}{3}$.

c. Here's how you could have solved the last equation:

$$\frac{2}{3}x + \frac{5}{2} = \frac{8}{3}$$

$$\frac{4}{6}x + \frac{15}{6} = \frac{16}{6}$$

$$4x + 15 = 16$$

$$4x = 1$$

$$x = \frac{1}{4}$$

Now consider the equation $\frac{8}{x} + 3 = 7$. Before we solve, name one value that x could not be equal to.

d. We know right away that $x \neq 0$ because this would result in a division by 0. Now solve $\frac{8}{x} + 3 = 7$. ***Hint:*** Use a similar approach to the method you used on the last two equations.

e. Here's how you could have solved the last equation

$$\frac{8}{x} + 3 = 7$$

$$\frac{8}{x} + \frac{3x}{x} = \frac{7x}{x}$$

$$8 + 3x = 7x$$

$$8 = 4x$$

$$x = 2$$

Now use the same type of approach to solve the equation: $\frac{2x+7}{x-2} + 1 = \frac{11}{x-2}$. *Hint:* Be careful! What value of x would result in a division by zero?

In the last problem, you should have found that the equation $\dfrac{2x+7}{x-2}+1=\dfrac{11}{x-2}$ had **no solutions**. Here's the solution:

$$\frac{2x+7}{x-2}+1=\frac{11}{x-2}$$

$$\frac{2x+7}{x-2}+\frac{x-2}{x-2}=\frac{11}{x-2}$$

$$2x+7+x-2=11$$

$$3x+5=11$$

$$3x=6$$

$$x=2$$

But x can't be equal to 2 because that would result in a division by zero in the original equation! So there are no solutions.

We say that $x=2$ is an **extraneous solution**. An extraneous solution is an incorrect answer we get despite doing correct algebra. Can you see why we might have picked up an extraneous solution in the last equation?

2. Consider the following equation: $\dfrac{x-2}{x+3}+\dfrac{2}{x}=\dfrac{2x+21}{x^2+3x}$

   a. This equation involves three fractions. Our first step is to find their common denominator. Write the common denominator in factored form.

   b. You should have found that the common denominator was $x(x+3)$. Now identify the values of x that would result in a division by zero in the original equation? ***Note:*** These will be important in identifying extraneous solutions.

c. You should have found that $x \neq 0$ and $x \neq -3$. Now solve the equation
$$\frac{x-2}{x+3} + \frac{2}{x} = \frac{2x+21}{x^2+3x}$$

Solution:

$$\frac{x-2}{x+3} + \frac{2}{x} = \frac{2x+21}{x^2+3x}$$

$$\frac{x(x-2)}{x(x+3)} + \frac{2(x+3)}{x(x+3)} = \frac{2x+21}{x(x+3)}$$

$$x(x-2) + 2(x+3) = 2x+21$$

$$x^2 - 2x + 2x + 6 = 2x + 21$$

$$x^2 + 6 = 2x + 21$$

$$x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

$$x = 5 \text{ or } x = -3$$

But $x \neq -3$ as we saw in part **b**. So the only valid solution is $x = 5$.

3. Solve each of the following equations. Find all solutions. Be careful! Some problems have extraneous solutions. *Note:* Answers are provided at the end of this problem.

a. $\dfrac{3}{x} + \dfrac{1}{3} = 1$

b. $\dfrac{2}{x+1} - 3 = \dfrac{1}{2}$

c. $\dfrac{3}{x} + 1 = \dfrac{-9}{x-12}$

d. $\dfrac{x+3}{x+1} = \dfrac{4}{x-2}$

e. $\dfrac{2}{x+1} + 1 = \dfrac{3}{x-2}$

f. $\dfrac{5x}{x-2} - 7 = \dfrac{10}{x-2}$

g. $\dfrac{x+5}{x-3} + 1 = \dfrac{30}{x^2-9}$

h. $\dfrac{10}{x} - \dfrac{6}{x+1} = 1$

i. $\dfrac{2x-3}{x-1} = \dfrac{x+1}{x+2}$

j. $2x - \dfrac{x}{3} = \dfrac{5}{x}$

k. $\dfrac{20+0.5x}{30+x} = 0.6$

l. $\dfrac{3x}{x^2-1} = \dfrac{5}{x+1}$

Answers a. 4.5 b. -3/7 c. 6, -6 d. 5, -2 e. $1 \pm \sqrt{10}$ f. no solution (2 is extraneous)

g. 2, -6 h. 5, -2 i. $\dfrac{-1 \pm \sqrt{21}}{2}$ j. $\pm \sqrt{3}$ k. 20 l. 2.5 (-1 is extraneous)