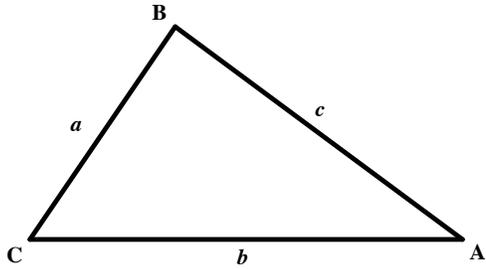


Algebra 2/Pre-Calculus

Law of Sines (Day 2, Inverse and Geometric Trig)

Name _____

1. Consider the diagram below.



- a. Prove that the area of the triangle is $\frac{1}{2}ab\sin C$. *Hint:* Start by finding the height.

- b. Explain why $\frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$.

c. Show that $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. **Note:** This relationship is called the Law of Sines.

d. Explain why $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. **Note:** This is another way of writing the Law of Sines.

Solutions

a. Let h be the height of the triangle (using side AC as the base). Then $\sin C = \frac{h}{a}$, so $h = a \sin C$. Thus, Area = $\frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(b)(a \sin C) = \frac{1}{2}ab \sin C$.

b. These are all the same relationship, just using different angles.

c. We prove this algebraically. (See below.)

$$\frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C$$

$$bc \sin A = ac \sin B = ab \sin C$$

$$\frac{bc \sin A}{abc} = \frac{ac \sin B}{abc} = \frac{ab \sin C}{abc}$$

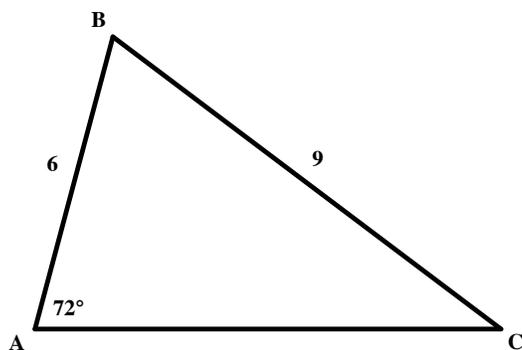
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

d. If each of these quantities are equal, then their reciprocals are equal as well

Use the Law of Sines to answer each of the remaining questions.

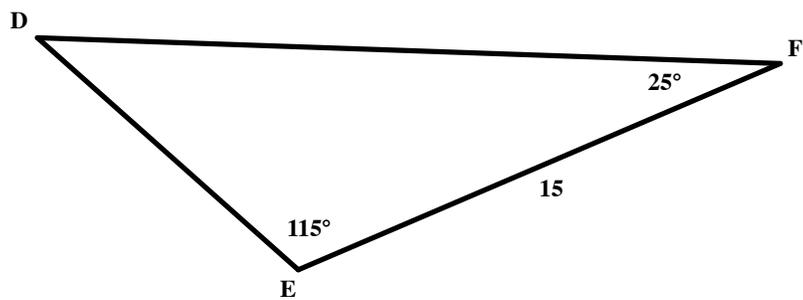
$$\text{Law of Sines: In } \triangle ABC, \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

2. Find all missing sides and angles for the triangle below.



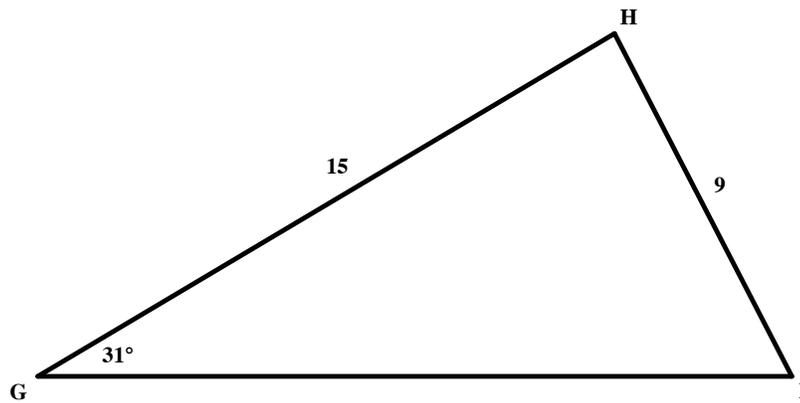
Answers $\angle C = 39.35^\circ$, $\angle B = 68.65^\circ$, $b = 8.81$

3. Find all missing sides and angles for the triangle below.



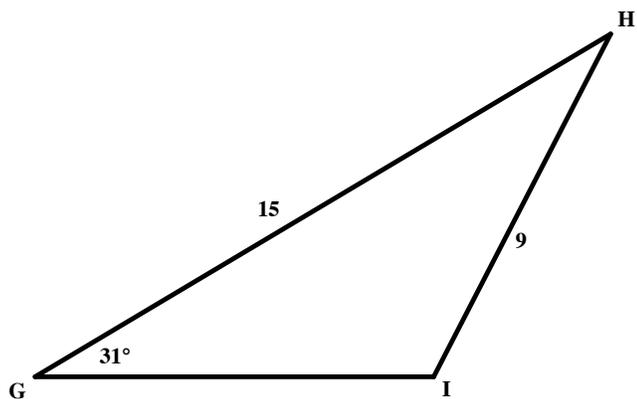
Answers $\angle D = 40^\circ$, $e = 21.15$, $f = 9.86$

4. Find all missing sides and angles for the triangle below.



Answers $\angle I = 59.14^\circ$, $\angle H = 89.86^\circ$, $h = 17.47$

5. Find all missing sides and angles for the triangle below.



Answers $\angle I = 120.86^\circ$ (You should have found that $\frac{\sin 31^\circ}{9} = \frac{\sin I}{15}$, so $\sin I = 0.8583$. But the equation $\sin I = 0.8583$ has two solutions. Make sure you know how to find both of them!) $\angle H = 28.14^\circ$, $h = 8.24$

6. Look back at the last two problems. Notice that in each problem we were given the same information about $\triangle GHI$. Specifically, we were told that $\angle G = 31^\circ$, $GH = 15$, and $HI = 9$. But this information generated two different triangles, one in which $\angle I$ was acute and one in which $\angle I$ was obtuse.

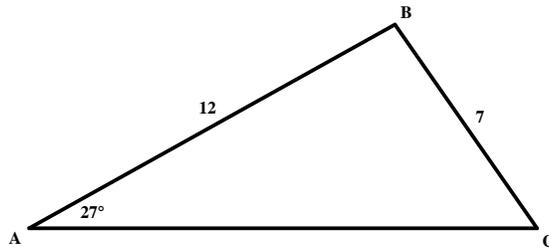
Consider the following information about $\triangle ABC$: $\angle A = 27^\circ$, $c = 12$, $a = 7$.

- a. Draw two different possible versions of $\triangle ABC$. **Note:** The pictures are given on the next page. But try to figure it out on your own first!

- b. Find all missing sides and angles for each version of $\triangle ABC$. **Note:** Answers are provided on the next page.

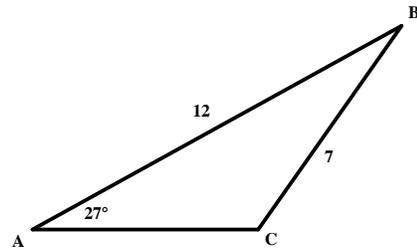
Answers for Problem 6

Version 1



$\angle C = 51.10^\circ, \angle B = 101.90^\circ, b = 15.09$

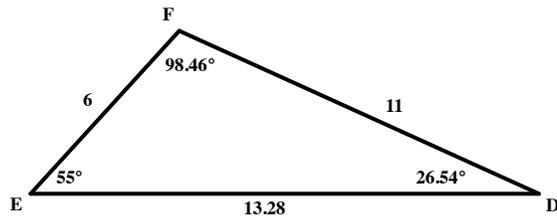
Version 2



$\angle C = 128.90^\circ, \angle B = 24.1^\circ, b = 6.30$

7. Consider the following information about $\triangle DEF$: $\angle E = 55^\circ$, $d = 6$, $e = 11$. Does this information result in one triangle? Two triangles? No triangles? Draw it. If more than one triangle is formed, draw both of them. Then find all missing sides and angles. **Note:** The answer is provided on the next page.

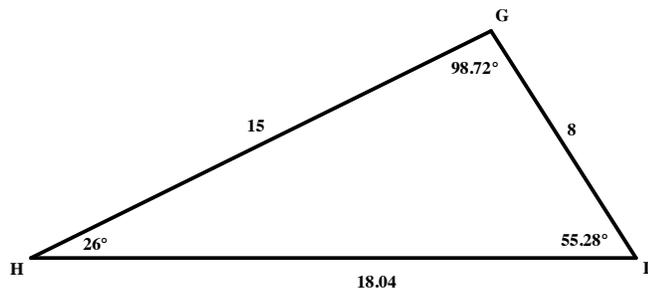
Answer for problem 7 Only one triangle formed:



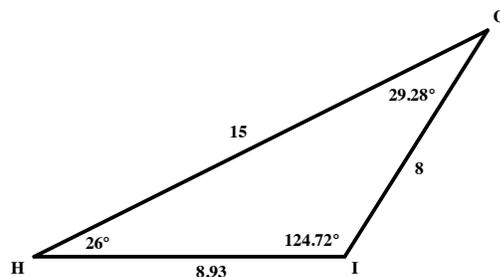
8. Consider the following information about $\triangle GHI$: $\angle H = 26^\circ$, $i = 15$, $h = 8$. Does this information result in one triangle? Two triangles? No triangles? Draw it. If more than one triangle is formed, draw both of them. Then find all missing sides and angles. **Note:** The answer is provided on the next page.

Answers for Problem 8 Two triangles are formed:

Version 1



Version 2



9. Consider the following information about $\triangle ABC$: $\angle A = 72^\circ$, $c = 10$, $a = 3$. Does this information result in one triangle? Two triangles? No triangles? Explain. **Note:** The answer is provided below.

Answer The Law of Sines tells us that $\frac{\sin 72^\circ}{3} = \frac{\sin C}{10}$. But this yields the equation $\sin C = 3.17$, which has no solution. So no triangles are formed.

Here's a way to think about this problem: Since $a = 3$, that side isn't "long enough to reach the other side." Here's how it would look if we tried to sketch it:

