Algebra 2/Pre-Calculus

Name_____

Trigonometric Identities (Trigonometry, Day 5)

In this handout we will continue using the circular trig definitions (x, y, and r, rather than adjacent, opposite, and hypotenuse). All of the problems on this handout should be done **without the aid of a calculator**.

1. Prove the following identity: $\cos^2\theta + \sin^2\theta = 1$. Your proof should include a diagram.

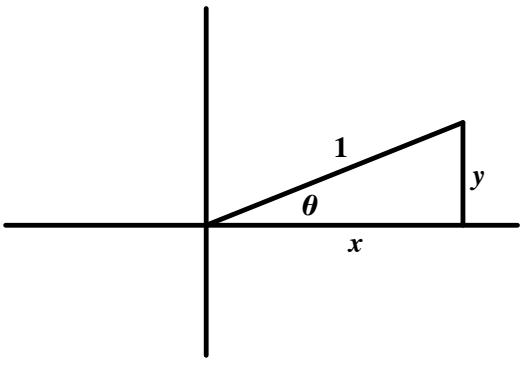
Solution

Start by drawing a diagram with $r = 1$ (as shown to the right). For any angle θ, we know that $x^2 + y^2 = 1$. And since $r = 1$, we also know that $x = \cos\theta$ and $y = \sin\theta$. Thus, by substitution,

$$x^2 + y^2 = 1$$

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

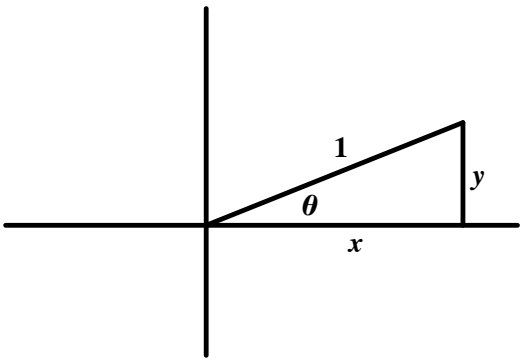
$$\cos^2\theta + \sin^2\theta = 1$$



2. Prove the following identity: $\tan\theta = \frac{\sin\theta}{\cos\theta}$. Your proof should include a diagram.

Solution

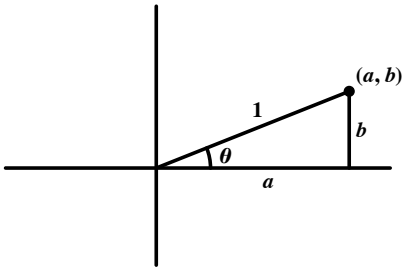
Start by drawing a diagram with $r = 1$ (as shown to the right). We know by definition that $\tan\theta = \frac{y}{x}$. And since $r = 1$, we know that $x = \cos\theta$ and $y = \sin\theta$. Thus, by substitution, $\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$.



3. The goal of this problem is to find an identity for $\sin(\theta+180)$.

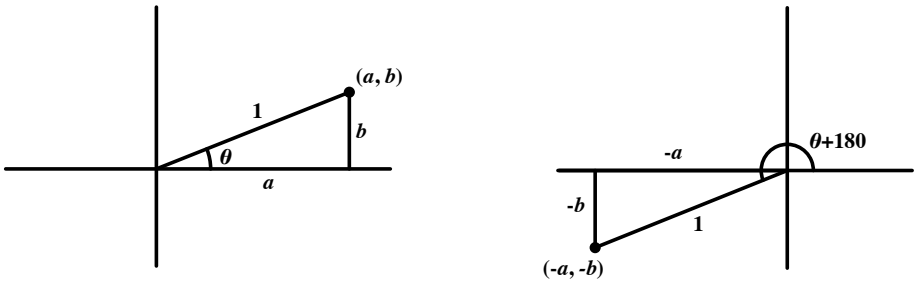
a. Is $\sin(\theta+180)$ equal to $\sin\theta$, $\cos\theta$, $-\sin\theta$, or $-\cos\theta$? Explain how you know.
Note: If you need a hint, skip ahead to part **b**.

b. Let's begin by drawing a triangle with $r=1$ in quadrant 1. Since we don't know the other sides of the triangle, we will simply label them *a* and *b* as shown in the diagram below.



We now must consider the angle $\theta+180$. Which quadrant is this angle in? Can you draw it? What are the side lengths? Are any sides negative?

c. Pictured below are the two triangles you should have made in the part **b**.



Can you use these pictures to find the value of $\sin(\theta+180)$?

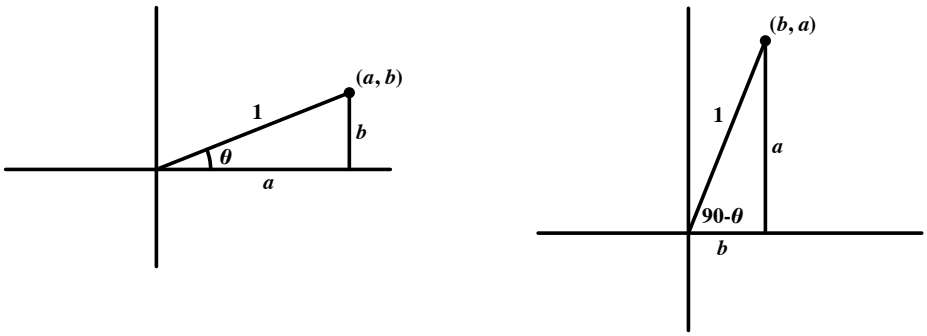
d. You should have found that $\sin(\theta+180) = -b = -\sin\theta$. Now use the diagrams to determine whether $\cos(\theta+180)$ is equal to $\sin\theta$, $\cos\theta$, $-\sin\theta$, or $-\cos\theta$.

Answer d. $\cos(\theta+180) = -\cos\theta$

4. The goal of this problem is to find an identity for $\cos(90-\theta)$.

a. Is $\cos(90-\theta)$ equal to $\sin\theta$, $\cos\theta$, $-\sin\theta$, or $-\cos\theta$? Explain how you know.
Note: You should start by drawing two triangles, just as you did in the in the last problem.

b. You should have found that $\cos(90-\theta)=\sin\theta$, as shown in the diagrams below.



What is an identity for $\sin(90-\theta)$?

c. Optional Challenge What is an identity for $\tan(90-\theta)$?

Answers b. $\sin(90-\theta)=\cos\theta$ c. $\tan(90-\theta)=\cot\theta$

5. Determine whether each of the following expressions is equal to $\sin\theta$, $\cos\theta$, $-\sin\theta$, or $-\cos\theta$. Draw a diagram for each part, just as you did in the previous problems. Make sure it is clear how you got your answer. ***Note:*** Answers are provided at the end of this problem.

a. $\sin(180-\theta)$

b. $\cos(180-\theta)$

c. $\sin(\theta+90)$

d. $\cos(\theta+90)$

e. $\sin(-\theta)$

f. $\cos(-\theta)$

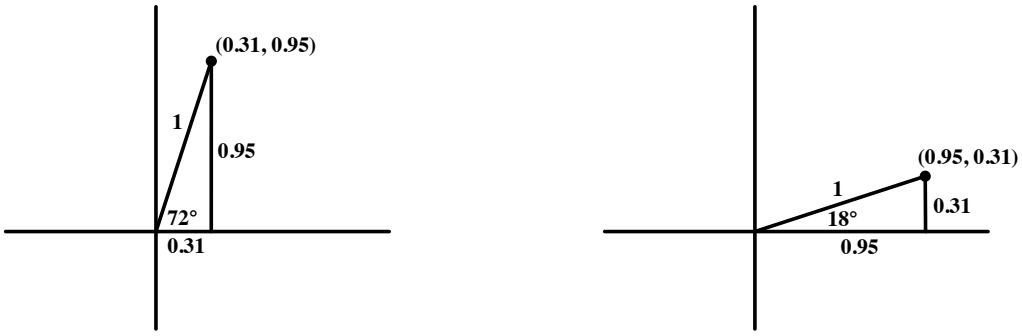
Answers a. $\sin\theta$ b. $-\cos\theta$ c. $\cos\theta$ d. $-\sin\theta$ e. $-\sin\theta$ f. $\cos\theta$

6. Suppose you are told that $\cos 72^\circ = 0.31$. (This is an approximate value.) Find each of the following.

a. Find $\sin 72^\circ$. ***Note:*** You may use the square root feature on your calculator, but do not use the trigonometric functions.

b. You should have found that $\sin 72^\circ = 0.95$. Now find $\cos 18^\circ$. Do not use your calculator.

c. To solve the last problem, draw two triangles, one with an angle of 72° and one with an angle of 18°. (See the diagram below.) Use the diagram to determine that $\cos 18^\circ = 0.95$.



Now use a similar approach to find $\cos 252^\circ$. Do not use your calculator. ***Note:*** Answers are provided at the end of this problem.

d. Find $\sin 108^\circ$. Do not use your calculator.

e. Find $\sin(-72^\circ)$. Do not use your calculator.

f. Find $\cos 108^\circ$. Do not use your calculator.

g. Find $\cos(-72^\circ)$. Do not use your calculator.

h. Find $\cos(-18^\circ)$. Do not use your calculator.

i. Find $\sin(432^\circ)$. Do not use your calculator.

j. Find $\sin(288^\circ)$. Do not use your calculator.

Answers c. -0.31 d. 0.95 e. -0.95 f. -0.31 g. 0.31 h. 0.95 i. 0.95 j. -0.95